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OCCASIONING CHANGE IN MATHEMATICS TEACHERS' BELIEFS -

by

CORINNE MCCABE

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ABSTRACT

The purpose of this study was to explore the impact of a series of workshops, based on the enactivist perspective, on the attitudes and beliefs of a group of practicing mathematics teachers. The intent of the workshops was to provide opportunities for teachers to examine and reflect on their beliefs about the nature of mathematics and the teaching and learning of mathematics.

Twenty six mathematics teachers, from both rural and urban boards, participated in a series of four workshops between November 1997 and April 1998. The implementation of the Western Canadian Protocol in Alberta schools in the fall of 1998 served as a focal point for our discussions. Each session was designed to provide an occasion for the participants to learn about the mathematics in the protocol and to reflect on their own pedagogical beliefs relating to mathematics teaching. The approach encouraged the teachers to bring their experiences to the situation and to have a role in determining the nature of the final outcomes of the workshop. The action research design enabled each successive session to be guided by the results and input from previous sessions. Data collected included an initial survey on teacher attitudes and practices, numerous taped discussions in which teachers reflected on their mathematics teaching practices and a final survey of teacher feedback on the impact of the workshops.

This approach proved to be a very effective format for teacher inservice. The dual nature of teacher and learner assumed by participants and facilitators created a learning environment that was constantly emerging. The attention paid to dealing

with implementation issues early in each meeting ensured that the teachers were able to focus on becoming reflective practitioners. There was a clear indication that these teachers have a will to approach the teaching of mathematics in a more open and diverse manner and that, in fact, in many of their classrooms significant changes in their practices have already occurred. The need and desire for ongoing support and inservice in both teaching methods and mathematical topics was evident in discussions and comments. Although participants identified many factors that would impact the amount of change that would occur in their classrooms the key issues that continued to emerge were time and resources - both human and financial. Many of the participants felt strongly that the opportunity to share ideas and reflect on practices and approaches would serve as a powerful agent of change.

This project has implications for inservice providers, curriculum developers, university preservice programs and classrooms teachers. Mathematics teachers, regardless of their experience and expertise, require ongoing occasions for learning if they are to move their teaching practices in a direction that is more flexible and designed to have students become critical problem solvers.

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Occasioning Change in Mathematics Teachers' Beliefs

Chapter 1 The Beginning

Developing the Question

The first time I encountered the words "curriculum revision" as a classroom teacher I was struck with horror. After 5 years of teaching, I finally had a handle on what I needed to teach, the students I was teaching and how I could combine the two successfully. I had gone through the beginning years of experimenting with different approaches and finding some that were very successful and others that, for no apparent reason, failed miserably. The suggestion that I would have to start from scratch was both frightening and disheartening. When the final revisions arrived I was pleasantly surprised to find mostly familiar topics and the reassurance that I did not have to completely start over. The introduction of new textbooks gave a new face to the mathematics courses, but the content, with the exception of a few new topics, was familiar and the pedagogy was consistent with how I had learned as a student and how I had been teaching.

Now, close to twenty years and several curriculum revisions later, my perspective is considerably different. I was glad the Senior High Mathematics courses were scheduled for an Alberta government review as part of their regular review of the scope, sequence, and content of all courses. That this review coincided with the development of the Western Canadian Protocol in Mathematics [protocol] was not a coincidence. This inter-provincial protocol formed the basis for the revisions to the mathematics programs from kindergarten through grade 12. In my varied roles as

classroom teacher, high school administrator, diploma examination manager and university student, I looked to the protocol to serve as the agent of change that was needed in many areas. Over the past several years many areas of mathematics education have been the source of increasing concern. The questions that occupied my thinking were numerous and came from a variety of perspectives. These included:

- Would this curriculum change address the problems we were encountering with the students who had not experienced success in junior high school and had come to high school convinced they could not and were not interested in doing mathematics?
- Would this revision bring us closer to meeting the standards set out in the early 1990s by the National Council of Mathematics Teachers [NCTM] which encouraged us to get all students involved in mathematics by allowing them to communicate their mathematical understandings, by problem solving and by using technology to broaden their mathematical horizons?
- Would the Western Canadian Protocol, as proposed, offer teachers windows and doors into a revised understanding of mathematical learning as proposed by researchers?
- How large would the role of technology be as a teaching and learning tool?
- Were teachers ready to make a change in their pedagogical beliefs about what it means to teach and learn mathematics?

•What support or inservice would assist teachers in becoming familiar with the Western Canadian Protocol , its philosophy and the implications of its implementation to their classroom?

•Would the Western Canadian Protocol lead to real change in classroom practices or would it result mainly in a rearrangement of the topics and the introduction of a few new content areas?

The focus of all of these questions seemed to me to lie in a larger more all-encompassing question. Thus after much wondering and reflecting on the nature of change my research question evolved:

How does one create occasions that encourage teachers to reflect on and make changes to their pedagogical content knowledge?

The Rationale for the Research

With this question in mind I began an in depth exploration of several areas. A close examination of the documents related to the protocol and the NCTM standards for professional teaching and evaluation, combined with research on both the constructivist and the enactivist perspectives about learning and cognition and the nature of the change process, brought me to the belief that the proposed changes will move mathematics education in a direction that, as an educator I believe, will be beneficial for all. What emerges from the Western Canadian Protocol could place mathematics teaching and learning on a new playing field. The Protocol questions the role of using predominately procedural and algorithmic methods as teaching tools. It clearly indicates that, the days of the mathematics classroom with students, a teacher, a blackboard, books, and chalk are changed. Technology is a requirement of the

course and not an added extra. Algebraic skills are being redefined. Mathematics is set in the physical and economic world and presents students with problems to solve using technological tools and their natural curiosity. Ideally, the new program redefines the role of teachers as a fellow learner with their students. The teacher's role becomes one of providing opportunities for mathematical learning and serving as a guide and a resource to their students.

Although much discussion about the nature of students and the nature of learning in mathematics has taken place at universities, in research programs, at conventions and in the professional journals, converting these abstract words and ideas into action in the classroom will not be a simple task. Teacher beliefs and practices are well entrenched and after years of successful classroom teaching these will not be discarded easily. Creating, in the minds of teachers, both a need for and a desire to try a new approach to the teaching of mathematics is an essential first step. I believe this can best be achieved by involving teachers in opportunities to study the proposed programs and their rationale and to reflect on their own pedagogical beliefs and teaching experiences. In the 1994 NCTM yearbook, *Professional Development of Teachers of Mathematics* it suggests that these opportunities are best provided away from the rush and chaos of the school day and in a setting that encourages open discussion and discovery. The purpose of this study is to explore the extent to which , through the proposed workshops, that establishing an occasion for this type of atmosphere among peers in a non-threatening atmosphere, prior to the implementation of new curriculum, may foster a more positive attitude toward change and instill a willingness to experiment with different approaches to teaching and learning mathematics. If this curriculum change is to result in a real shift in the

paradigms for the teaching of mathematics, then teachers need to be exposed to occasions where they, themselves, are asked to learn in a less traditional manner. These situations have to result in a successful learning experience for the teacher if they are to be viewed as effective.

The Historical Perspective

In order to understand the magnitude of the proposed changes to the mathematics curriculum and the mindset of the teachers involved in making the changes it is important to have a historical perspective of mathematics education in Alberta in the 1990s and the development and philosophy of the Western Canadian Protocol. It is these factors that have created a climate in the mathematics teaching community that is conducive to true change.

The last change to the *Alberta Senior High Program of Studies in Mathematics* took place in 1990 shortly after the release of the National Council of Mathematics Teachers standards on curriculum and assessment . These standards were revolutionary in nature in that they asked educators to reexamine the purpose of mathematics education and the role of mathematics in our society. The three factors that were most critical in driving these change were:

1. the changing nature of the discipline of mathematics;
2. the changes in society's emphasis on mathematics and;
3. the changes in our understanding of teaching and learning mathematics.

The NCTM document presented a reform agenda for mathematics education and a vision of appropriate mathematical goals for all students. *The Curriculum and Evaluation Standards*, released in 1989 focused not only on content priorities and

emphasis but also provided a direction for how learning should occur and be evaluated. The standards offered a double edged sword of both opportunity and challenge to mathematics teachers, schools and school divisions in terms of reshaping content, pedagogy and assessment. In 1989 *The Curriculum and Evaluation Standards for School Mathematics* and in 1991 *The Professional Standards for Teaching Mathematics* were released. These documents were designed to provide a framework from which mathematical reform could be built and to provide teacher with ideas on how to move towards teaching that focuses on:

- worthwhile mathematical tasks
- the teacher's role in discourse
- the student's role in discourse
- tools for enhancing discourse
- the learning environment
- analysis of teaching and
- the process of evaluation (3 standards)
- the foci of evaluation (5 standards)

(NCTM, 1991)

The 1990 *Alberta Program of Studies for Mathematics* attempted to take up the spirit of these standards. It talked about the importance of communicating in mathematics and about using technology to assist in the teaching and learning of mathematical concepts but, with the exception of a few units, did little to support the development of a vision of a new kind of mathematics. On one hand, teachers were encouraged to use a graphing tool, either a computer program or a graphing

calculator, to assist students in problem solving, on the other hand they were assured that no student would be required to use this technology on a diploma examination. In fact, all diploma examination questions would be guaranteed to be "technology neutral". The curriculum encouraged teachers to have their students get involved in problem solving and to use authentic real world examples in their lessons. Yet most of the topics remained the same. Traditional algebraic topics that could effectively be taught using algorithmic and procedural approaches continued to be taught in this manner. The magnitude of the content demands of the courses provided little time for the exploration that the changes were encouraging. Therefore, although the majority of teachers made some attempt at introducing some technology and problem solving into their mathematics classes it was done on a limited basis. In the best interest of their students, they stayed with proven methods that would ensure student success. Problem solving remained relatively traditional and the structure of many mathematics classes continued to be predictable.

The Western Canadian Protocol

The Western Canadian Protocol in Mathematics (1996) shares many of the purposes and underlying philosophies found in the NCTM standards in particular with regards to beliefs about the nature and role of mathematics, the nature of the learner and the growing role of technology as a learning tool. It was initiated in 1988 when the six Western Canadian Provinces and Territories agreed to work cooperatively to develop a common curriculum. This cooperative undertaking to rethink the mathematics curriculum was agreed to because each of the participating provinces and territories believed in the importance of common educational goals; high

standards in education, removing obstacles for student access to educational opportunities (which includes improving the ease of transfer from jurisdiction to jurisdiction); and optimum use of educational resources (Western Canadian Protocol, June 1996, pg. 1).

The project began in 1991 with an initial informal review of all provincial programs that revealed that the these programs were not substantially different. A more formal review by the directors of curriculum from across Western Canada, held in 1993, confirmed these findings. Mathematics across the provinces covered the same basic topics, just in a different order and with slightly different emphases. Shortly after this review, development of the protocol in mathematics began for students from kindergarten through grade 12. Work on the framework was completed in 1996 and implementation details were left to the individual provinces. (Implementation schedule for Alberta -Appendix A)

The beliefs embedded in the protocol about the nature of cognition and learning are very similar to those held by educational theorists who are referred to as constructivists and enactivists. The belief that students must attach their own meaning to the mathematics they learn is central to the philosophy of the program. Learning is best developed when occasions are provided that gradually move the mathematics that students experience from the simple to the complex and from the concrete to the abstract. The protocol addresses the need for teachers to be aware of different learning styles and developmental stages of the students and to use a wide variety of pedagogical methods so that the students might develop solid mathematical concepts that they can apply to a variety of problem solving situations. All of these mathematical learning situations must be structured in such a way as to value the

input of each individual student and encourage risk taking (Western Canadian Protocol, 1996, pg. 2.). The protocol establishes a conceptual framework that weaves skills, procedures and concepts together based on a set of mathematical processes and the nature of mathematics. The framework is organized by grade and strand and each area contains general and specific outcomes and illustrative examples.

The goals of mathematics education for all children as described in the preamble of the protocol are:

- using mathematics confidently to solve problems
- communicating and reasoning mathematically
- appreciating and valuing mathematics
- committing themselves to lifelong learning
- becoming mathematically literate adults, using mathematics to contribute to society
- creating a positive attitude towards mathematics (Western Canadian Protocol, 1996, p.3)

The mathematical processes described in the protocol define goals for mathematical learning that could easily be identified as the needs of any lifelong learner or the activities that should be built into any learning environment. They include the skills of communication, estimation and mental mathematics, reasoning, visualization, problem solving, making connections and using technology appropriately. Each of these skills uses different types of learning and encourages the involvement of the whole student and all of their senses. Students are not limited to listening and writing but are encouraged to get involved actively in their learning and

to take advantage of oral communication skills, group dynamics and kinesthetic strengths to facilitate their learning. Students are encouraged to go out on a limb and be risk takers. This is the kind of activity that is important for all learners, be they students of mathematics or mathematics teachers struggling with a new paradigm about the nature of mathematical learning.

In contrast to learning mathematics instrumentally and out of context, the type of learning proposed by the protocol encourages students to see the relationship of mathematics to a wide range of topics, to look for connections between subjects and to explore connections to their daily life. It would be encouraging to see students who could see the connections of the mathematics they learn, outside the classroom. Any teacher can relate stories of students who compartmentalize knowledge to such an extent that today's math lesson is forgotten before they arrive at the physics class next block. (This is particularly frustrating to the teacher who has taught them math and followed them down the hall to teach them physics.) By establishing learning situations that put mathematics in context and by working in a collaborative manner to create interdisciplinary experiences, students can begin to see these connections. To a large extent the highly technological world that we live in has done much to bring students to a better and broader understanding of the interdependent nature of their world.

The rapid development of and ready access to technology has greatly increased the daily use of a wide variety of methods to organize and process data. The widespread use of presentation programs, spreadsheets and graphing programs to display data has greatly expanded individual's ability to communicate clearly and

effectively. Incorporating the use of these technologies is a critical component of a program designed to prepare students for the 21st century.

The structure of the protocol at the senior high level is such that for each grade there are two distinct approaches to mathematics and a common core that have been identified as essential mathematics for all students. Each of the approaches emphasizes a different aspect of mathematics. The Pure Mathematics Program is algebraic and graphical in nature whereas the Applied Mathematics Program is numerical and geometrical in its focus. The pure approach is directed to students who will continue in studies that require the rigorous use of algebra and calculus and in many ways is very similar in content to the current Mathematics 10-20-30 programs. The Applied Program on the other hand is very different in nature. It is focused on the use of mathematics as a tool for problem solving and there is very little explicit traditional algebra outlined in the specific learning outcomes. Rather these topics are embedded in the learning outcomes that focus on more generic problem solving skills. Within each approach and the common core at each grade level are specific learning outcomes that encompass the four mathematical strands that have been identified as the organizing principles for the protocol. These strands are number, patterns and relations, shape and space, and statistics and probability.

A Change in Perspective—Approaches as Opposed to Streams

One of the most dramatic differences between the current programs in Alberta and the protocol as it will be implemented in Alberta is the elimination of the concept of streaming. The Applied Program is not intended as a second stream of mathematics for weaker students but rather it is intended for students whose

mathematical needs lie in a more practical direction. This approach to program development parallels the science programs where students are encouraged to chose one or more disciplines after their grade 10 year of general science. In mathematics, the grade 9 year of the protocol introduces the students to formal algebra as well as graphical, geometrical and numerical methods. Students at the grade 10 level are then given the opportunity to choose the course that best fits their learning style and future needs. Explaining this change in philosophy to all of the stakeholders in the educational community is one of the major tasks facing teachers and the program implementation teams. Although many of the concepts and ideas in the Applied Program originate directly from the input of the business community, there exists considerable skepticism about how well this type of learning will be accepted as mathematical and whether it will attract the intended audience. The inclusion of specific learning outcomes that require the extensive use of graphing technology, matrices and spreadsheets as tools for problem solving must be seen as providing new opportunities for mathematical learning. Providing students with the opportunity to gain the ability to explore interesting and practical applications that would otherwise be beyond their reach must be viewed as a positive step for mathematics education. Not only must students, parents and teachers be convinced that the mathematics contained in the Applied Program is important mathematics that will serve the needs of graduating students, but the post secondary institutions must be willing to reevaluate what they will accept as their entrance requirements—particularly in areas that do not require calculus.

One of the ironies of the arguments regarding the appropriateness of the Applied Program for high school students is that, on the one hand, it is argued that it

is not rigorous enough and does not contain enough traditional algebra, yet when teachers examine the specific objectives they express concern that many of their current Math 13 students will not be able to "handle" the level of mathematics required by the learning outcomes. There is little doubt in my mind, that there is a small group of students whose previous experiences in mathematics will be such that they will not be prepared to handle either the Pure or the Applied Mathematics Program at the high school level and the current Math 14-24 program will need to be revised to meet their needs. However, personal experience has convinced me that many of the students who do not experience success in the current Math 13 program fail to do so not through lack of ability but because of their own belief that they cannot or will not do mathematics. This is a belief that is founded in their previous failures. For many of these students the Applied Program, if properly implemented, may offer the practical focus that mathematics has lacked that will motivate them to try again.

This phenomenon was illustrated to me vividly in a conversation I had with a colleague who was struggling with the level of cooperation he was receiving from his Math 13 students. We were discussing how frustrating it was to get them on task and keep them there when he remarked "I was subbing in the Career and Technologies Studies (CTS) class the other day and not only were these same kids hard at work and fully engaged in what they were doing, but they really seemed to be enjoying themselves. How can I hope to compete with that?" It is in the Applied Program where there is the greatest chance to see real change in the way mathematics teachers view what it means to teach mathematics. It is here there are real opportunities to create a new paradigm for students mathematical learning.

Perhaps, a new approach to mathematics can put mathematics classes on an even playing field with other areas of the students' schooling—one that is seen as more practical and more enticing. Perhaps, if the students can take the skills they have learned in their CTS classes and elsewhere and apply them to mathematical problem solving, they will begin to see mathematics as part of their everyday life and not as an isolated subject that is difficult for them.

The Design and Purpose of the Research

Motivation for the Study

In September 1998 Mathematics teachers across the province of Alberta began implementing the Western Canadian Protocol for Mathematics at the senior high level. The protocol as a whole, and the Applied program in particular, presents a program that has as a major focus applying mathematics to everyday situations and problems. The Applied program is numerical and geometric in nature and is not focused on learning procedural mathematics nor algebraic manipulation. These are new directions for both students and teachers.

The Western Canadian Protocol for Mathematics represents a major shift in the approach to teaching and learning mathematics both in the content taught and how it is taught. The philosophy of the protocol includes the belief that students must learn mathematics by doing mathematics, embedded in contexts, in a meaningful way. This learning must provide students with opportunities that allow them to explore mathematical ideas and grow in their mathematical understanding. The methods that are used to achieve these goals must promote active learning, participatory knowledge and interdisciplinary approaches. The emphasis is on conceptual based mathematics

that encompasses modeling, analysis of data, and the use of logic. The instructor's role becomes one of creating occasions for mathematical learning and providing access to the tools required for doing real mathematics (Field, 1993). Making connections to the real world and personalizing the mathematical experience are all central to the this new paradigm for mathematical instruction.

With the stage set for radical change it is essential that teachers be provided with the support they need to implement change both in the curriculum they teach and the way they teach it. Many mathematics teachers have an overriding need to keep everything under control, to set the agenda and to ensure things proceed as planned. Their scientific deterministic training makes it difficult for them to focus on the need to practice listening, to guide reflection, to find ways of observing their teaching and to accept the students' views and directions as fundamental aspects of the classroom dialogue. Successful implementation of the new program depends on a careful plan, with an inservice component designed to assist teachers in examining, reflecting, listening to their students and revising their beliefs about the nature of mathematical learning and teaching. The following description of the research study I undertook outlines the structure and findings of the series of workshops that were a first step in a process aimed at meeting this need.

Research Site

The purpose of this study was to explore how I could create occasions that encouraged teachers to reflect on and make changes to their pedagogical content knowledge. The intent was to have teachers examine their practices and beliefs in light of the upcoming changes to the curriculum. In the process we would examine

the barriers to change and through discussions and actions, begin to work towards learning situations that encouraged open ended problem solving and mathematical discourse.

With these goals in mind I set out to establish a cohort group of mathematics teachers with whom I could engage in action research. I had created in my mind a series of workshops, modeled on the enactivists beliefs on the way in which learning is best facilitated, which used the specific learning outcomes of the new mathematics curriculum as a content focus. By creating an environment that would provide teachers with an opportunity for their own learning as well as one for reflection on current practices and linking the workshops to the imminent changes in the curriculum I hoped that there would be sufficient incentive for teachers to become involved. Recruiting a group of classroom teachers who would be willing to participate in such a program became my first task.

I initially was approached by the Learning Network, a group of professional educators responsible for providing inservice opportunities to teachers in the Edmonton area, in May 1997 to discuss a series of workshops for high school science teachers on performance assessments. I used this opportunity to discuss my desire to involve a group of mathematics teachers in a workshop on creating mathematical learning environments. After some discussion it was agreed to offer a program through Continuing Professional Education that teachers could register for by using their professional development credits or by paying through school or personal funds. The initial flyers went out from the Learning Network in September 1997 advertising a series of four workshops to be held between November 1997 and March 1998 entitled *Improving Student Performance - Creating Occasions for Mathematical*

Learning. The initial description of the course focused on using alternate assessment and teaching techniques in the classroom as a method for improving student performance. Although there were a few initial registrations I was concerned about obtaining sufficient interest to complete my study. To encourage more people to consider attending, I approached the loosely organized coalition of teachers from the schools that serve the communities surrounding the city of Edmonton. This group consists of actively involved mathematics teachers who meet periodically throughout the year to discuss issues of concern in mathematics education. When I spoke to them about my proposed workshops they expressed interest, particularly if we could tie the activities to the implementation of the new curriculum. I already felt strongly that the change to the curriculum was going to be the catalyst needed to encourage teachers to examine their attitudes and beliefs about the nature of mathematics teaching and learning so I assured them that this was my intended direction. I had previously done an extensive investigation into the types of activities that could be used to involve students in their own learning of mathematics. Many of these activities came from NCTM publications, magazines designed for mathematical educators or the internet. In addition I had spent a considerable amount of time becoming familiar with the philosophy and intentions of the protocol. I planned to share this preliminary research with teachers during the workshops. Based on the interest expressed by this group, I was certain I would have sufficient registrations to proceed with the workshops and therefore began to plan my workshops and the related research.

In the planning stages I concentrated on general themes and the processes I would use to present the ideas. Each of the workshops was to focus on one particular aspect of teaching and learning mathematics and to relate this to the protocol. In my

initial proposal I carefully outlined the intended outcomes for each of the four sessions. The four broad categories initially outlined and a brief overview of my objectives are described below:

1. Exploring the Philosophy and Content - Involve teachers in a discussion of the structure, philosophy and content of the protocol and relate it to the NCTM standards. Review literature on the enactivist perspective with a view to discussing this at the next class as it relates to the philosophy of the protocol.

2. Experiencing Activities that Create Learning Situations - Involve teachers in a series of activities, related to the protocol, that are open ended in nature. Discuss how these model enactivist teaching. Reflect on the implications of this type of activities for mathematical learning in the classroom.

3. Creating Occasions for Learning - Engage teachers in designing an open ended performance assessment to be used in their classroom that relates to the protocol. Have teachers use the activity they design in their classroom and be prepared to share student work and how they assessed it with fellow participants at the next class. Reflect on the pros and cons of this type of activity.

4. Investigating Alternate Assessment - Share the activities designed by the participants and their results with the workshop participants. Focus and reflect on how we can assess open ended activities in the classroom. Investigate various scoring rubrics for their suitability in assessing cooperative and covergent learning.

Although I began with these plans, it quickly became evident that my broad based goals were far too ambitious and would require significant modification. It was important for me remain flexible in my planning and to ground each future session on the information I had gathered from the previous session. I needed to be able and

willing to adjust the agenda to meet the needs of the participants. Each session served to set a direction for me both for reading and planning for future sessions. This was one of the advantages of choosing action research as a research paradigm. It allowed both action and research to take place simultaneously because it allowed the researcher to adapt to the changing situation. The reflective spiral of this type of research facilitated the integration of theory and practice and provided a direction for the next move.

The Research Methodology

The first step I needed to take was to gain some insight into the background and experience of the individuals I would be dealing with. In addition, I needed to have a perspective on their beliefs and values with respect to teaching mathematics. So, in early October, I began research on the kinds of questionnaires that had been administered on teacher beliefs about mathematics and mathematics teaching and learning. There was certainly no shortage of examples. These ranged from the very broad based surveys completed by teachers in the Third International Mathematics and Sciences Study (TIMMS), to local surveys measuring the results of a particular inservice activity. Surveys existed that studied student performance, teacher performance, use of manipulatives, use of computers and calculators, teacher job satisfaction, issues impacting student performance and a wide variety of other important topics. It became apparent that I was going to have to narrow the scope of the survey if I wanted to ensure that the responses were true representations of teachers beliefs. Reports on existing surveys indicated difficulties with both the level of return and consistency of responses if the length was excessive. I decided to focus

on four areas and to limit the questions to five questions per area. Although the survey was initially intended to be administered before the first class meeting, time and registrations procedures prevented this. A copy of the survey is included as Appendix B and the results from the twenty respondents who completed this survey are summarized in Appendix D.

I also collected teachers' reactions to the methods I used in the workshops as well as their emergent beliefs about teaching styles for the classroom as we explored alternate activities and the new curriculum. The data collected includes audio tapes of some of the discussions from each of the sessions. In session one the exploration activity was taped for each group. The reflections on this activity were taped in session two. In addition to these reflection tapes, selected groups were taped as they participated in the open ended activities. In the final session the presentations by the participants were taped. The quality of the tapes is marginal because of background noise but transcripts were made of several tapes and generalizations and direct quotes based on the transcripts and the tapes are used throughout the thesis as support for conclusions.

Field notes taken by myself, as the group facilitator, and impressions as provided by research colleagues also served to provide valuable data in analyzing the results of this project. Particularly useful were the written reflections I engaged in at the end of each session and the informal interviews conducted with the participants and colleagues both during and after the sessions. These opportunities for reflection and discussion not only served to help in planning future sessions but served to bring meaning and coherence to each experience. "It is often said that we learn from experience, but in fact we do not often learn from experience alone. Something more

is required" (Mason, 1997). It is the reflections and connections that we gain from our experiences that provide meaning to them. It is by discussing the experiences and reflecting on them that we are able to connect them with other like experiences and thus we can "label" them for future use.

Another useful source of data proved to be the activities that the participants designed. The choice of activity combined with the comments made in presenting it helped to establish the participant's perspective at the end of the sessions. This data combined with the data obtained from the final post-session survey, which was completed by twelve participants, helped to provide a collective view of the participants' views with respect to proposed changes, mathematics education and the value of the inservice project. The final survey was intentionally open ended in nature to provide participants with an opportunity to reflect on their mathematics teaching and their learning. A copy of the final survey is included as Appendix C and the results are included in Appendix E.

The Results of the Research

The following chapters of my thesis consist of a synthesis of my experiences. They take the form of a study of the reactions of this cohort group, how these actions helped to define future directions for the workshops, the outcomes of the series of workshops, and my reflections on the lessons learned from this experience as well as suggestions for future directions for creating opportunities for mathematical learning for both teachers and students. The data analysis will use multiple sources of information to look at the possible outcomes of the study in terms of: action research as a research approach for working with teacher inservice; the potential of this type of

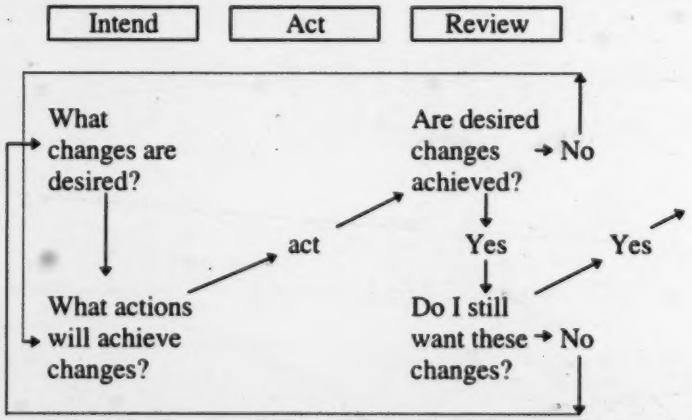
activity for facilitating teacher reflectiveness and change; the enactivist perspective as a model for learning; and factors influencing teachers' acceptance of new ideas.

This type of teacher inservice represents one possible approach to entering into discussion with mathematics teachers on pedagogical change and curriculum reform—perhaps as a result of this experience other formats or possibilities may emerge. One side benefit of this exploration will be to help delineate how one might undertake an exploration of the relationships and strategies involved in the teaching and learning processes and how these might best be reported so as to be useful to others undertaking their own explorations.

Action Research as a Research Paradigm

The action research cycle can be viewed as a learning cycle. It employs systematic reflection as a critical component of both the action and the research. It creates a partnership between the researcher and the participants that is both effective and practical. There is little if any gap between theory and research practice and it is responsive to the emerging situation. As with all forms of action research much of the nature and content of the actual workshops in my study evolved with the input of the participants. Whenever research is designed to improve the quality of what goes on in the classroom, it is the practitioners of that action that define the direction that it must go. Any attempt to make education fit strictly into the scientific movement has had little long term effect on what actually happens in classrooms. It has long been argued that it is only the students of education and not its practitioners who choose to adopt the scientific method to study student and teacher behavior (Corey, 1953). The practitioners of education recognize that the model of hypothesis, procedure, and

observation leading to a final conclusion is often not feasible. There are too many detours and intervening factors along the way. Education is not painted black and white but is a mural full of gray areas where there are no definite boundaries. However, this is not to say that teachers are not interested in becoming involved in a process that looks at their problems scientifically in an attempt to help them make solid evaluations and decisions that will benefit students. In fact, teachers are naturally drawn to action research. Its cyclical nature suits the type of problems and dilemmas they face. One must decide on the problem to be studied, narrow the focus of the problem, clarify and research any uncertainties within the problem statement, detail a plan of action, carry out the plan, observe, record and draw conclusions from actions taken, reformulate the plan and begin again.



Intend–Act–Review Cycle for Action Research (adapted from Dick, 1998)

At all times the plan must be flexible and capable of revision if it is to be sensitive to the input of the participants. The opportunity to grab a "teachable moment", listen to an insightful comment or go in a new direction must not be lost. Changes are much more likely to occur if they are founded in an inquiry initiated by a teacher than by a top down mandated agenda. It is the very definition of action

research, as a process, that makes the firm definition of anticipated outcomes impossible. In many ways the proposed series of workshops could be compared to a spiral of steps, each of which is composed of planning and acting that results from the actions and reactions of participants in the previous session (Kemmis and McTaggert 1990).

Curt Lewin argued that in order to "understand and change certain social practices, social scientists have to include practitioners from the real world in all phases of inquiry" (McKernan 1991, p.10). Regardless of the definition of action research one ascribes to there are four basic themes that must be present: empowerment of participants, collaboration through participation, acquisition of knowledge and social change (Zuber-Skerrit 1991, p.2). This study encompasses all four of these themes.

Limitations of the Study

The purpose of the study was create occasions for teachers to examine their beliefs about the nature of teaching and learning mathematics as well as to examine how being involved in the workshops occasions reflection on these beliefs and practices. It makes no attempt to measure actual change in classroom behavior. These workshops were a first step that encouraged teachers to become reflective about their practices before being faced with the incredible work load of implementing new courses. In addition, the sessions were designed to create a mindset for doing things differently and hopefully would avoid becoming entwined in issues of implementation. The research is focused on one particular group of teachers. There is no control group of teachers with which to compare attitudes towards classroom

techniques and the philosophy of the protocol. This comparison, would in itself, be an interesting future study. The absence of a control group will decrease the generalizability of the study. The initial survey on teacher beliefs was administered anonymously and so it was impossible to examine change in beliefs or attitudes over the course of the five month period. In addition, the survey is one I have designed and so the results will not be compared to other groups. No data were collected regarding the reasons individuals chose to become involved in the study. These underlying reasons for participation may influence motivation for change. All participants were volunteers and had paid to participate in a professional development activity and this lead to a biased in the sample. The group cannot be considered representative of all teachers.

Delimitations of the Study

The group consisted of teachers from a predominately large urban area and its immediate surroundings. Most of the members were from larger schools where some degree of collegiality existed and sharing of ideas was relatively commonplace. Teacher beliefs and practices were assessed on the basis of a self assessment and not on data from students, parents or colleagues. One of the delimitations was my dual role as researcher and facilitator. As facilitator it was very difficult to keep accurate field notes. I attempted to minimize this difficulty by taping the proceedings, recruiting research assistants, and making class notes immediately after sessions. Another factor that impacted the study was inherent in the nature of action research. Although the general question may have been clear at the outset of the study, it had "fuzzy edges" because to some extent the participants and the data gathered in early

stages served to further determine the next steps in the process. The responsiveness of action research to the participants input allowed the study to redefine itself more clearly for each cycle (Dick, 1998).

An interesting perspective of this action research was it's dual nature. As we progressed through the workshops, action research was happening on two levels. There was my research as a professional development provider, attempting to engage teachers in inservice designed to facilitate change in their teaching beliefs and practices and there was each individual teachers' research into their own teaching practices. In each class we entered with a set of *intentions* based on our own experiences, were involved collectively in a set of *actions*, and participated in group and individual *reflections* on what these meant to us. From these reflections, emerged our collective and individual intentions for future behaviors in our teaching situations.

Chapter 2 Examining Teaching from A New Perspective

The literature review presents a synopsis of selected literature and research on the critical considerations that are involved in the process of examining, reflecting on and changing mathematics teachers' beliefs and practices. These considerations include the nature of beliefs about: how students learn; mathematics as a discipline; how mathematics is learned and taught; the process of change; how change can be facilitated through teacher inservice; and selected initiatives designed to support change in mathematics classrooms.

Beliefs about the Nature of Learning

Behaviorists

Traditional beliefs and practices related to the nature of learning are based on the behaviorist paradigm. Here, the learners are believed to be "blank slates" on which the teacher engraves knowledge. This system of learning implies that knowledge is transmitted by the teacher to the students and the way in which the students organize this knowledge in their minds is a reflection of the order that exists in the world (Bruner, 1985). Behaviorists believe that there exists a knowable reality and that learning can be objective and factual. Students are exposed, in an inductivist manner, to a series of experiences that lead them to the truth about mathematics. For a true behaviorist, learners acquire knowledge through experience and through their senses but they do not actively manipulate this knowledge to form their own perceptions. Thus conceptual differences do not exist between students, since the teacher has transmitted the truth about the mathematical world as it exists.

Constructivists

Over the past several years there has been extensive discussion and research based on the constructivist philosophy of education. This movement is based on cognitive views of learning as opposed to behaviorist views. Constructivists believe that the organization of learning is based on internal representations of reality. They believe that learners are active participants in the learning process and that their prior learning, belief systems and previous experiences will determine how the new knowledge is assimilated. Since each student brings his own perception to the knowledge, conceptual differences will exist among students being presented with the same situation (Carter-Cohn, 1993).

At the current time, there are several schools of thought that support the constructivist approach in varying degrees. These have been categorized by Paul Ernest (1992) to be the trivial, radical and social constructivists. The trivial constructivists argue that students suffer from misconceptions when they fail to understand the true nature of the things. Geddis (1991) indicates how this reflects the "traditional view of teaching ... that holds central the concept of knowledge as justified, true beliefs" (p.4). This is contrasted by the radical and social constructivists who believe that there is no absolute way to view the world. They believe that individuals filter the incoming knowledge in such a way that they create their own reality and that there is no access to the world except through their individual perceptions. Thus, since reality is unique for each individual, it is not possible for reality to exist as a common group conception. If one is to accept the constructivist position, then learning must be seen as a rational activity in which students take the presented information and internalize it in a manner that is

consistent with their personal organization of reality. This would suggest that learning is akin to informed decision making and that the outcomes of learning will be different for each individual.

Enactivists

The enactivists propose a view of cognition and learning based on the theories of Bateson, Maturana, Rosch, Thomson and Varela (1991). Enactivism is an embodied theory of mind. In the enactivist world we see not survival of the fittest but survival of the fit. In this world individuals and their environments are a unit that are codetermined. The individual has an "organization" that allows an observer to distinguish it from any other individuals, however this organization is a product of the observer (Reid, 1996). The world the individual exists in does not have a real definition but rather, it is defined in terms of the individual's perception of reality. Perception and action are inseparable. Just as genes are interdependent, so are individuals and the world they live in. It is the structure of each individual's system that determines what happens to them and the actions of an outsider only trigger this response. Thus perception, action and cognition are all functions of the structure. From this perspective, telling a student that mixing two chemicals will produce gold is only instructive to the extent that the student knows what "mixing is" and understands that attaining gold is desirable otherwise these instructions will not trigger the student to take action.

This is not to say that the system is static. Change is always a possibility for the system, but only changes that will allow the system to continue to act adequately are viewed as cognition or learning. A system like this, that is open to constant

change, that is self-referential, self-organizing and self-producing is referred to as an autopoetic system. It is the openness to change and to self redefinition that makes this system a learning system.

Enactivists believe that changes in the environment and the learner's system are both determined by the structure of the system. The system has a limited set of actions it can undertake to make any change in its structure. This is referred to as structural determinism. Meaning resides in a series of complex patterns of activity that emerge from the interactions of the system and the surroundings. Both the system and the environment are free to interact and as long as this interaction continues to occur they are said to be structurally coupled and coemergent (Varela, Thomson and Rosch, 1991).

From the enactivist perspective knowledge does not preexist in any form but it is enacted in a particular situation. Action is perceptually guided; learners select the stimuli in the learning situation to which they respond. Enactivists study how the learner guides his actions in a particular situation. The way in which the learner embodies the world, and not the world itself, determines how the learner can act and be changed by events. The learner and the environment mutually specify each other. Thus, if one is to accept the enactivist view of learning and cognition, classroom learning situations must be structured to create opportunities for structural change.

Conceptions about the Nature of Mathematics and Mathematics Teaching

Much research has been done over the years on the conceptions teachers have on the nature of mathematics, the teaching and learning of mathematics and on the impact of these belief systems on classroom practice.

A teacher's conception of what constitutes mathematical knowledge is a blend of learned beliefs, both conscious and subconscious, preferences that are often a reflection of the teacher's learning style, and a philosophy of learning based largely on classroom experiences. Although there are numerous studies that attempt to categorize teacher conceptions, one of the most effective was done by Ernest in 1988. The three conceptions of mathematics described in his study have a basis in the philosophy of mathematics (Benacerral & Putman, 1964; Davis & Herch 1980; Lakatos ,1976 as cited in Grouws, 1992) and have also been documented in empirical studies of mathematics teachers. Ernest (1991) described the three conceptions of mathematics as follows:

> First, of all, there is a dynamic, problem-driven view of mathematics as a continually expanding field of human creation and invention, in which patterns are generated and then distilled into knowledge. Thus mathematics is a process of enquiry and coming to know, adding to the sum of knowledge. Mathematics is not a finished product, for its results remain open to revision (the problem-solving view).
>
> Secondly, there is the view of mathematics as a static but unified body of knowledge, a crystalline realm of interconnecting structures and truths, bound together by filaments of logic and meaning. Thus mathematics is a monolith, a static immutable product. Mathematics is discovered, not created (the Platonist view).
>
> Thirdly, there is the view that mathematics, like a bag of tools, is made up of an accumulation of facts, rules and skills to be used by the trained artisan skillfully in pursuance of some external end. Thus

mathematics is a set of unrelated but utilitarian rules and facts (the instrumentalist view). (p.10)

To suggest that each teacher's beliefs can be exclusively assigned to one of these categories would be an oversimplification. However, in her study of junior high mathematics teachers Thompson (1984), found a great deal of consistency between the teacher's beliefs and classroom practices and the conception of mathematics that are ascribed to them.

Another categorization of the conceptions of mathematics teachers is presented by Lerman (1983). He suggests that the philosophical basis of teachers' conceptions falls into one of two categories—the absolutist or the fallibilist. In the absolutist paradigm, mathematics is seen as being Euclidean in nature—absolute, value free, and only connected to the real world in an abstract fashion. On the other hand, the fallibilist perspective is one that sees mathematics as quasi-empirical in nature - based on conjecture, where proof and falsification are used as tools and uncertainly is seen as a natural occurrence . Based on these descriptors, Lerman studied the teaching styles of teachers and found a direct correlation between the teachers' conceptions of mathematics and teaching styles (Lerman as cited in Grouws, 1992).

William Perry (1970) developed a nine point scale for categorizing the level of intellectual and ethical development of college students based on their conceptions of what constitutes knowledge. This scheme has been adapted by a number of researchers (Copes, 1979, 1982; Stonewater & Oprea, 1988 and others as cited in Grouws, 1992) to study the conceptions of what mathematics teachers believe constitutes mathematical knowledge. In his study Copes (1979) identified four stages

of mathematical conceptions; absolutism, multiplism, relativism and dynamism and he related each of these to a period in the historical development of mathematics. Copes then studied a variety of teaching styles and found a direct relationship between teaching styles and conceptions of mathematics.

One of the strongest proponents of the relationship between teacher conceptions of mathematics and teaching style is Richard Skemp. In his work Skemp (1978) clearly identifies two conceptual frameworks: relational mathematics and instrumental mathematics. He characterizes relational mathematics teaching as using conceptual structures to find several solutions to a problem and instrumental mathematics teaching as rules without reason which will enable a student to get from a given starting point to a predefined and predictable endpoint. He summarizes his realization that, among students and mathematics teachers, there are proponents of both of these perspectives:

> We are not talking about better or worse teaching of mathematics ... It has taken me some time to realize that this is not the case. I used to think that math teachers were all teaching the same subject, some doing it better than others. I now believe that there are two effectively different subjects being taught under the same name "mathematics" (p.11).

Regardless of the conceptual beliefs held by an individual teacher, research on teacher beliefs has shown that there is a great deal of internal consistency between the teacher's beliefs about the nature of learning, the nature of mathematics and mathematics learning and the methods of instruction. Thus, if a change in teaching

practices is to occur, there must be an accompanying change in teachers' conceptions about the nature of learning in general and the nature of mathematics.

The Process of Conceptual Change

The process of conceptual change parallels the process of change in scientific beliefs. The literature on scientific change suggest that there are two periods that are characteristic of scientific activity. These are generally referred to as the period of normal science and the period of scientific revolution (Kuhn, 1970). In normal times when students, teachers and scientists are faced with a new phenomenon they rely on their current concepts to organize information and deal with these new concepts. Through the process of assimilation, they use existing concepts to account for the new phenomena. This is a relatively stress-free activity where students, teachers and scientists can ask sufficient questions, of the right type, to enable them to see the new phenomena as a new piece of the existing conceptual framework, consistent with their current beliefs. However, when these learners experience a new concept that does not fit with their current perceptions of reality and no amount of questioning will result in assimilation, a conceptual crisis occurs. A major shift must occur in their thinking process or the new concept must be rejected. This is the moment in the learners' time that parallels the time of scientific revolution. In order to accommodate the cognitive dissonance experienced by the learners a "conceptual revolution" must occur. New paradigms must be found that can explain and deal with the incongruous phenomenon.

For example, Posner, Strike, Hewson, & Gertzog (1982) suggest that there are four stages of change that must occur if conceptual change is to result. These stages

remain the same whether one operates from the constructivist perspective or the enactivist perspective. The first requirement for change is a dissatisfaction with the existing concept. A need for a better answer or explanation must exist or change is unlikely to occur. In order to create dissatisfaction with the existing concept one needs to create a situation which causes dissonance.

The introduction of a new curriculum has for many teachers created this state of uncertainty. Many teachers see clearly the inconsistencies between the ways they currently operate in their classrooms and the ways in which the new curriculum is designed to be implemented. What many teachers are uncertain of is why the proposed change is viewed as a better way of teaching mathematics. What problems will it solve or will it just create new ones? What these teachers lack is the second requirement for change—an understanding of why the new paradigm makes sense. They need to see how this new approach will solve their dilemmas and how it will lead to increased learning for all students. This understanding needs to be followed by circumstances that meet the third required condition for change— the new conceptions must be plausible. This perspective of teaching mathematics must be seen as something that can be practically instituted in teachers' classrooms. It must be manageable. It must be seen as able to do all the things the current methodologies do, as well as to deal with those that are not working within the current framework. The final condition that must be met if a paradigm shift is to occur is that the new concept must have the potential to lead to new knowledge. It must suggest new directions that may extend knowledge and new directions that can be explored. It must be seen as an opportunity to improve learning.

The Learning Cycle

If an inservice is to be successful, in helping teachers to embrace the new philosophy of the Western Canadian Protocol and the NCTM standards and to change their beliefs and practices, then it should attempt to lead the participants through the four stages of the learning cycle (Fleming & McPhan, 1993). These stages closely parallel the stages of change described in the preceding section and can be characterized as

1. Examining
2. Questioning
3. Acting
4. Reflecting

In the preliminary stage of the learning cycle the teachers need to examine their current belief systems and conceptual bases. They need to reflect on their current practices and teaching strategies. This will lead to a second phase which is a time of questioning for the teachers. Here, they must ask which outcomes of the new curriculum and which aspects of the standards cannot be addressed with their current framework. At this point, they are faced with readily observable problems that cannot be managed within the existing set of rules for classroom instruction. As they move into the next stage of the learning cycle, the learners are encouraged to become actively involved in a wide range of activities designed to help them visualize how the new concept might be used to explain many of the same ideas their current practices do well in addition to dealing with some of the troublesome problems the current paradigm could not solve. The final stage is a time of reflection for the learner. It is a time for the teacher to internalize the new concept and to consider its implications on

other areas of learning. It is a time for the teachers to reflect on their personal beliefs and how learning occurs and how teaching can best encourage this.

I believe the key factor in developing any successful learning environment is starting from where the learner is at and setting up experiences that will occasion reflection and problem solving. Whether change is viewed as a result of making adjustments to the existing internal mental structure or as a change in the relationship between the person and his environment, or both, it must be change that comes from within and not change that is externally imposed - if it is to be lasting change. The process of change, for change really is a process and not a product, may result in adding new information, restructuring existing information or deleting ideas that have proven inadequate. The structural nature of the change leads to a stable and unified conceptualization of reality. That this conceptualization of reality may differ for different individuals leads to conceptual diversity. This is not a negative outcome as it is often useful to have different rules for different people in different situations.

Research on Professional Development of Mathematics Teachers

In his article in the 1994 NCTM Yearbook, *Professional Development for Teachers of Mathematics*, Doug Clarke summarizes the research on the professional development and lists ten key principles. These are listed below and will be referred to in more depth in the analysis of the teacher inservice plan described later. A successful teacher inservice or staff development plan must:

1. Address issues of concern and interest, largely (but not exclusively) by the teachers themselves, and involve a degree of choice for participants.

2. Involve groups of teachers rather than individuals from a number of schools, and enlist the support of the school and district administration, students, parents and the broader school community.
3. Recognize, and address the many impediments to teachers' growth at the individual, school and district level.
4. Use teachers as participants in classroom activities or students in real situations, model desired classroom approaches during inservice sessions to project a clearer vision of the proposed changes.
5. Solicit teachers' conscious commitment to participate actively in the professional development sessions and to undertake required reading and classroom tasks, appropriately adopted for their own classroom.
6. Recognize that changes in teachers' beliefs about teaching and learning are derived largely from classroom practice; as a result such changes will follow the opportunity to validate, through observing positive student learning, information supplied by professional development programs.
7. Allow time and opportunities for planning, reflection, and feedback in order to report successes and failures to the group, to share "the wisdom of practice" and to discuss problems and solutions regarding individual students and new teaching approaches.
8. Enable participating teachers to gain a substantial degree of ownership by their involvement in decision making and by being regarded as true partners in the change process.

9. Recognize that change is gradual, difficult and often painful process, and afford opportunities for ongoing support from peers and critical friends.
10. Encourage participants to set further goals for their professional growth (p.38).

Research in Support of the Reform Movement in Mathematics Education

Extensive research has been completed since the National Council of Teachers of Mathematics issued *The Curriculum and Evaluation Standards for School Mathematics* (NCTM, 1989) and *The Professional Standards for Teaching Mathematics* (NCTM, 1991). Teachers and researchers have been involved in a wide variety of projects designed to assist in the implementation of the NCTM standards. In a series of articles contained in two volumes of *What's Happening in Math Class*, edited by Deborah Shifter (1996), teachers who have been part of the "Summer Math for Teachers" program share their classroom experiences as they attempt to embrace a new paradigm for teaching mathematics. This program, and many similar ones, have as their underlying premise that teachers must critically reflect on their own beliefs, the nature of mathematics and the teaching and learning mathematics as a starting point for exploring new directions and strategies for their teaching. This reflection is a prerequisite to any work that needs to be done on analyzing how students learn or exploring how topics from the curriculum can best be addressed (Shifter, 1996). In the vignettes included in these books two main topics are addressed—Envisioning New Practices and Reconstructing Professional Identities. In each story, a teacher shares a classroom experience and reflects on how it exemplifies one or more of the

aspects of mathematics teaching and learning or how it contributes to a new vision of what it is to be a mathematics teacher.

Another initiative that uses teachers' experiences, as a source of inspiration to change practices, is the 1996 NCTM Yearbook on *Communication in Mathematics*. In the preface Portia Elliot, the editor, makes the convincing point that the task of moving the United States to mathematical literacy is in the hands of classroom teachers. The subsequent articles that follow in the yearbook illustrate how teachers are achieving this by creating mathematical discourse communities in their classrooms.

The Atlanta Math Project (AMP Philosophy, 1997) is another example of a cooperative effort among teachers to initiate and integrate the standards, as outlined by the NCTM, into their philosophy of teaching. In this project, participants base their classroom experiences on the belief that "everyone should be a teacher and everyone should be a learner". The importance of listening to students as a way of understanding their problem solving strategies is stressed. The use of cooperative learning strategies, where students are encouraged to use each other as resources, is one aspect of this program that is particularity effective in engaging students in mathematical problem solving.

Virtually every volume of every periodical dealing with the teaching and learning of mathematics contains numerous articles on meeting the agenda for change, implementing the standards in the classroom, creating mathematical learning environments, using technology to teach mathematics, cooperative learning techniques and other related topics. Yet change is slow in coming. Finding a way of

connecting the classroom teacher to these articles and initiatives is one of the goals of the inservice program.

Conclusion

This purpose of this chapter was to provide an overview of the both current and historical perspectives and paradigms about the nature of learning in general and of mathematics teaching and learning in particular. It has reviewed theories of conceptual change, the learning cycle, the research on professional development and current initiatives in teacher education as necessary considerations in developing an inservice program designed to assist teachers in implementing a new curriculum that changes the focus of mathematics teaching.

In the following chapters, I will report on the evolution of a study designed to provide opportunities for change based on the enactivist perspective as a model for learning about mathematics learning and teaching. In this model the teacher is seen *in the middle* of a reciprocal activity (Convey, 1995, Simmt, 1996 as cited in Kieren 1996) in which the students and teachers learn from one another and the situation in which they exist. The actions of the students are not guided in any particular direction. The teacher sets up a *learning occasion* which allows a wide range of outcomes and then *listens* to and helps to shape possibilities based on the actions and discussions of the students. This shaping may lead in an intended direction or may branch off to some unexpected area of inquiry. Although it may often seem that each student is thinking and acting on their own, the actions of the group and the shaping provided by the teacher result in the group progressing toward some common, *covergent,* goal. The teacher in this situation must be constantly alert to the changing

perspectives of the students and must provide ways for these to become part of the "lesson".

The *proscriptive* nature of the evaluation of this type of classroom allows that any outcome that is not forbidden is allowed and valued. It is a learned skill for the teacher in this environment to set limits between boundaries that are too narrow to allow value to be assigned to all students' contributions and those that are too wide to ensure students are staying within the scope of the objectives. The enactivist perspective promotes a learning environment that resembles a complex network. Each participant's contributions play an integral role in weaving the learning outcomes and individual learning cannot be separated from the learning of others or the collective group. The development of a community of learners who learn with each other is a critical component of this model of learning. The learning of one student is often dependent upon the actions of others. It is from this type of learning that *worlds of mathematical significance emerge for all of the participants* (Kieren, 1996).

The following chart displays a comparison of the enactivist model to other models of mathematical teaching. It provides an overview of the differences in the approach of this model compared to other approaches to mathematical teaching.

	Teacher Place	Teacher Act	Listening	Listening Goal	Reasoning	Discourse	Goals	Evaluation
Direct Representation	In Front	Tell (TIRE)	For	To Check	Verify	Directed	Convergent (uniform)	Prescriptive
Emergent/ Constructivist	Over Shoulder	Facilitate Construct	To	To Portray	Explore	Self-discourse Parallel Play Teacher query	Divergent (individually justified) Personal Coeherence	Evaluative
Coemergent Enactivist	In the Middle	*Occasion *Listen	With	To Bring Forth a World of Mathematic al Significance	Explore / Explain	Interactive: *discourse of mathematical conversation	Co-vergent	Proscriptive

Figure 5 Patterns in Mathematics Teaching
Kieran, 1996. (used with permission)

It is the enactivist theory of learning around which, I believe, the Western Canadian Protocol for Mathematics is structured. The teacher inservice program , *Improving Student Learning - Creating Occasions for Mathematical Learning*, uses this perspective as a guiding principle for establishing tasks designed to facilitate teacher learning about themselves and their beliefs.

Chapter 3 The Workshop Experiences

Getting to Know the Participants

The first source of information I obtained to help me in both my planning of the workshops and my understanding of the participants' perspectives were the results of the pre-session surveys (Appendix D). These provided me with a wealth of information that served to provide background for the participants' comments and to inform my decision making.

Background Information

The first section of the survey was designed to help me in the preparation of the workshops. Since the survey was not completed until the first group meeting I had to plan the initial class based on informal discussions I had had with some of the participants. The information gained from this section of the survey confirmed that the participants included a broad range of high school teachers who taught mathematics to students at all levels. In addition, I had two teachers who also taught grade nine mathematics. This added an extra dimension to the plans I could make, as their experiences with the new program and with students who had been through the grade nine program would be a valuable aspect to integrate into the workshops. It was also clear from the background information that many of the participants had been teaching mathematics for many years and were in fact viewed as mathematical leaders in their school communities. The final makeup of the group included both new and experienced teachers, those trained as mathematics experts and those who

fell into teaching mathematics for a variety of reasons, teachers from large urban high schools and those from isolated rural communities.

The information gathered on the professional development activities that these teachers had been involved in provided valuable information about the strengths each could bring to the group discussions and activities. The wide scope of activities, including participation in national and international conferences, leadership activities and involvement in work with publishers and educational television, clearly indicated this was a talented group of dedicated teachers. This diversity would add an exciting dimension to our discussions over the next several months. The two common characteristics, that were exhibited by all the participants, were a genuine desire to find out more about the changes proposed and to discuss ways of connecting the mathematics to their students' lives in a real way.

The results on questions two and three about the participants level of awareness of the details of the philosophy of the Western Canadian Protocol, the details of the general and specific learner outcomes, and the NCTM standards on Curriculum and Evaluation were quite surprising. Of the twenty respondents seventeen indicated they were not familiar with the details of the philosophy, twelve indicated that they were not aware of the general and specific learning outcomes for the grades they taught and eighteen indicated they were unfamiliar with the details of the NCTM standards. Given that copies of the protocol had been distributed to all schools as much as one year earlier and that much of the philosophy of the protocol is based on the NCTM standards, this information represented a significant finding to me and would result in major changes to my plans.

In discussing these findings with coworkers in the Curriculum Branch at Alberta Education we felt we had identified a very real area of concern. If these teachers, who were self motivated and who had voluntarily registered for a workshop on improving student results, were not aware of the upcoming changes in curriculum and directions of mathematics education, then what was the larger picture? What was the level of awareness of the remainder of the teachers in the province with respect to the changes? This discussion added impetus to an implementation project that was being considered by Alberta Education - a province wide "road show" outlining important aspects of the protocol, the timelines for change and the technology requirements.

Nature of Mathematical Learning

This section of the survey was designed to help to establish which of the many perspectives of mathematics education best described the beliefs of these teachers. The results indicated that the vast majority of the respondents (19/20) strongly supported the belief that students learn mathematics best when it can be seen to be applicable to real life. There was also a high degree of support for the belief that all students can learn to think mathematically(16/20) and that problem solving should be a major focus of mathematics teaching (13/20). Participants were divided in their opinions of whether or not mastery of basics is a prerequisite to problem solving. This difference in opinions pointed to an area to probe in our future discussions.

When teachers were asked which of Ernest's three conceptions of mathematics they felt best described the nature of mathematics, fifteen of them selected the problem solving view, four selected the platonist view and none of the

participants selected the instrumentalist view. This information is particularly interesting when reflected on in conjunction with the emphasis these teachers placed on a variety of teaching styles as indicated in section three of the survey.

Teaching Strategies

Section three was designed to provide a picture of the types of activities that currently take place in the mathematics classrooms of participants. By awarding points to each response in the following manner I was able to establish a scale that allowed me to make comparisons of relative emphasis.

- 1 point was awarded for no emphasis
- 2 points were awarded for minimal emphasis and
- 3 points were awarded for significant emphasis

The resulting list is shown as Table 1:

TABLE 1 Relative Emphasis Placed on Classroom Activities

Activity	**Score**
Lecture	59
Discussion of Homework	55
Discussion	53
Seatwork	52
Students Using Graphing Tools	51
Tests/quizzes	48
Student reports/projects	43
Small group work	42
Performance Assessments	41
Students using computers	38
Students at the board	31
Research (library/internet)	31
Guest Speakers	22
Field trips	17

The most striking aspect of this list, to me, is that although the previous responses suggested that these teachers felt problem solving was the basis of the nature of mathematics, many of the activities used most frequently in their classroom are not those most often viewed as leading to effective problem solving. This

dichotomy seems to be in contradiction to the study performed with junior high teachers by Thompson in 1984 where she found a high correlation between teacher beliefs and their classroom practices. There are several possible reasons that could explain this apparent contradiction. Many of these teachers spend a significant percentage of their time preparing students for external exams either International Baccalaureate, Advanced Placement or Diploma examinations. The time constraints placed by curriculum content and these exams may well dictate some of the methods that receive increased emphasis. The reported emphases may well reflect the teachers' ways of dealing with large class sizes or limited resources both financial and human. It would be foolish not to recognize that many of these traditional methods have worked well in preparing students for further study in mathematics and even though teachers firmly believe there is more to mathematics education than imparting knowledge and teaching procedural learning, these skills go a long way in helping students to succeed on formal mathematics assessment. With a changing curriculum, the participants expressed hope that the evaluation that evolves will reflect the new philosophy and thus provide impetus for teaching that focuses more on problem solving and using one's day to day experiences to help develop a better understanding of the world of mathematics.

Participants' responses to other questions in this section do indicate that the participants value problem solving approaches to mathematical learning. The use of calculators and computers as a tool for teaching problem solving was seen as important by fourteen of the respondents. It is interesting to note, that although fourteen teachers indicated they use graphing tools a significant amount in their classrooms, only five teachers indicated a significant use of computers. The reasons

for this difference can be ascribed to the wide variety of circumstances particular to each teacher's individual situation. Discussions with participants and later activities revealed that for some, the limited use of computers was a result of computers not being readily available, while for others the problem was more a question of their own comfort level with incorporating computers in their instructional plans. Most teachers felt that because of the nature of the current Math 30 curriculum the use of a graphing calculator or a graphing program was an essential element in any complete program. There was strong support among the members of the group for activities that involved cooperative learning in small groups (17/20) as well as for involving students in writing about their mathematical ideas (14/20). There was a level of uncertainty about the value of activity based learning and performance assessments. Opinions were split as to whether the time and effort required to involve students in these types of activities was time and effort well spent. Some teachers expressed the view that this answer was very dependent on the nature of the course you were teaching and the degree to which time was a compelling factor.

Professional Growth

In a survey designed to provide increased insight into the group, I felt it was important to identify their views on this aspect of their professional life. In general teachers felt supported by their colleagues in trying new activities (14/20) and they valued the opportunity to share experiences, ideas and materials (15/20). The value this particular group of teachers placed on collegiality and sharing was no surprise to me, as I had recruited many of them for the workshops from their own informal support group. One of the primary factors for involvement for many in this group was

the opportunity to connect with other mathematics teachers and discuss issues and challenges. Another set of participants in the workshop represented an entire math department from a city high school. They had decided, or perhaps their department head had decided, to use the workshops as opportunities to meet and share experiences and ideas as they prepared to implement a new curriculum. The one member of their department who could not participate initially phoned me after the second class and asked to join us. She felt left out of the discussions back at school.

The one factor that was perceived to influence professional growth, that all of the participants expressed concern over at one time or another during our sessions and which was evident in this portion of the survey, was time. In the 1990s, time, or lack of it, seems to be one of the most frequently cited causes for many of the problems that impact education. Time for professional sharing, time for reflection, and time for experimentation are often the casualties of a too busy schedule. It is often easier and more expedient to rely on the known and the tested when time is at a premium.

One interesting, although not totally unexpected, result emerging from survey responses was the diverse opinions about the nature of inservice training teachers felt was important. There was a split in views as to whether it was more important to receive continual mathematical training or training in effective ways of teaching and managing mathematics classrooms. These findings raise some interesting issues for those designing inservice opportunities. In order to meet the needs of all teachers opportunities will have to be provided that allow them to improve both their mathematical skills and their repertoire of teaching ideas. The need for ongoing mathematics instruction reflects the teachers' desires to continue to upgrade and hone their mathematics and to maintain a high level of expertise in a field they enjoy. If the

teachers in this group are any indication, then teachers do not need to be encouraged to be lifelong learners. The introduction of new content areas to the Western Canadian Protocol has heightened this need for many teachers. But, teachers are also aware of their needs in terms of rethinking how we teach mathematics and how we get children engaged in learning in general.

The Starting Point - As Defined by The Responses of Participants

From the survey results and from the discussions I had with registrants it was clear I had the opportunity to work with a highly motivated group of teachers. Their needs included information on the philosophy and the specifics of the protocol as well as on how to implement it in a way that would engage students in their own learning within the realities of their school setting. Teachers hoped the workshops would help prepare them for the upcoming changes. My goal was to meet these needs by providing opportunities for them to find and organize occasions for mathematical learning - first for themselves and then for their students. My task was to work with these highly skilled professionals to provide them with opportunities to reflect on mathematics learning and their role in the classroom. Together we needed to examine how best to use the skills and experiences that formed our world in conjunction with those skills and experiences of our students to codetermine a mathematical world that is significant and meaningful.

Getting to Know the Western Canadian Protocol

Intentions

Our first workshop meeting was held on November 4, 1997. The objective of the class was to familiarize the participants with the philosophy, structure and contents of the protocol as well as to provide some background information on implementation plans and timelines.

In preparation for the session, I spent a considerable amount of time exploring ways of having the group "get their feet wet" in terms of the content of the program and in terms of approaching teaching in an enactivist manner. This was a diverse group and each of them brought their own set of internal perceptions and real life experiences to the situation. I wanted to capitalize on these and to provide opportunities for the individuals to interact with each other and with the materials provided. I believed, based on personal experience and readings I had done on the work of mathematics educators, that the participants' level of willingness to experiment with alternate delivery methods with their own students would be higher, if they themselves had been involved in a positive experience with a slightly different approach to learning and organizing information. I wanted to create an atmosphere that would allow the classes to evolve with the needs of the participants.

With these intentions in mind, I went into the first workshop with plans to spend a short time on introductions, philosophy and background and then have participants begin to explore the protocol through a jigsaw activity and then take part in an mathematical activity involving the use of manipulatives.

Implementation Issues

The class evolved very differently. My short introductions to the protocol and implementation plans took much longer than expected. But, the shared understanding gained by myself and the teachers in this first ninety minutes was extremely important. It quickly became evident that the issues that were uppermost in these teachers' minds were those related directly to implementation of the new curriculum and its impact on their classroom, department and school organization for the following year. Implementation dates had been announced for September 1998, but these teachers felt poorly informed. They had little information about resources, calculators and computer usage, appropriate placement of students in streams, post-secondary acceptance of the courses, availability of inservice for teachers and transfer information for students. Telling comments, illustrating the level of this frustration on the part of the participants, are found in the post-session survey, where in response to the question -

" What are your biggest areas of concern and how can these be addressed?"

one respondent commented

"the timing we have been given for implementation - if all the major decisions of implementation, textbook selection, etc would have been made ready a year ago..., then this set of workshops plus other inservices would have been much more productive - as we have spent more hours trying to find out what is happening, when it is happening, etc. - Most teachers are going to try to implement this curriculum with lots of baggage that they have had no control over."

Despite the considerable work I had done in preparation for the first session and my position as a staff member at Alberta Education there were many of their questions and concerns I could not readily answer. In fact, many of their questions were questions that I had asked myself. There would have been little value in limiting this discussion to the twenty to thirty minutes originally scheduled. It was clear that the sharing of concerns was providing an important outlet and creating a level of comfort among the group. My assurances that I shared their concerns and would go back to work and try to obtain answers helped to establish a level of trust. The participants needed to feel that someone from the government was listening to their concerns and would, at the very, least pass these concerns on to the appropriate individuals.

This discussion of implementation issues was to become the starting point of each of our sessions. It provided an opportunity for me to give updates on the current status of decision making and for them to share with each other how they were proceeding in preparing for the upcoming school year. I was also able to share their concerns and questions on an ongoing basis with the Curriculum Standards Branch staff and this input was valuable to them as they prepared information and inservices for teachers province wide. This was a good lesson for me in terms of setting the agenda for learning. As teachers we often push on with our own agenda guided by a desire to meet curriculum objectives and to ensure students learn what they "need to know." In doing so we shut the door to important learning. The learning that resulted from this dialogue and listening to each other had wide spread positive results for mathematics teachers across the province.

Making Sense of the Protocol

After I assured teachers we would continue this discussion at each of our sessions, they were prepared to move to the jigsaw activity. In this activity each of the small groups was asked to study one of the four content strands of the protocol across all three grades. After a period of time the class was regrouped so that each new group had one member from each of the strand groups. The task was for each member to "teach" all they had learned about their strand to the other members of the group. This type of jigsaw activity is common in the literature on cooperative learning and is suggested as an activity that promotes all group members becoming actively involved in contributing to the group discussion. The discussions that ensued were taped and many of the comments that follow are excerpted in whole or part from these tapes. Although the discussions on the tapes were difficult to follow because of background noise and long pauses where participants stopped to read or think about the documents, there are some key issues that emerged from the discussions.

The first broad category that emerged from the discussions was the need to make sense of the protocol document. Despite what I felt were very clear directions, groups had a difficult time getting started as they were not clear on the numbering system used to identify outcomes, nor where they aware of the organizational details of the document. With the exception of the brief overview I had provided, this was for many of the teachers their first exposure to the protocol document. This lack of familiarity was emphasized in many of the discussions on the tapes which carried an exchange similar to:

> "Applied should be the same as pure anyway in this part; that makes it common. So they are all together right now."

"Which one are you looking at, C5-3?

"Is that common or pure? How do you tell?"

"The C gives you the common the P the pure and the A the Applied"

In the course of their discussions important features of the curriculum gradually came to the forefront. In one group a discussion on volume led to the realization that unlike other mathematics curricula they had worked with this was not a spiral curriculum. Topics taught in one year, such as surface area, were assumed to be mastered and were not reintroduced in the next level. Rather, the discussion of area and volume began with new geometric shapes that had not previously been studied. Students were expected to form their own connections with previous learning.

The differences in the intended approach between the Pure and Applied math programs gradually emerged as the groups worked through the exercise. Although it took a bit of time for participants to get themselves oriented to the document, as they became more familiar with the structure they began to point out connections between clusters and approaches. On several occasions a participant can be heard on the tapes to comment that they have just seen a significant difference in the approach required for the different courses. The discussion in one group focussed on different skills required for solving systems:

"See this is it, because in common they have to graph linear systems but applied they have to solve the system of linear inequalities."

"So you have to know how to graph as a common skill before you can solve your system by graphing, right?"

"Everybody has to know how to graph, these guys have to be able to solve the system by graphs. And then your pure kids probably have to, the pure kids I'm guessing probably have to have to solve it algebraically."

"But look down here at the applied, it says solve algebraically ... ??"

"It looks like they kind of branch out, like they still have to do all this but yet they have to go even farther and do more.."

"More real life applications."

The jigsaw activity also served as a vehicle for teachers to share their concerns about the implementation of the new curriculum. Several discussions focused on the amount of planning and preparation that would be required. Teachers' concerns in this area stemmed from a number of different directions. Some were concerned about the need for change just "when I was getting a handle" on the current program. Others expressed concern about whether or not they had sufficient mathematical background to teach some of the new topics. More than once I heard the comment "I haven't done this (referring to a mathematical concept that was new to the curriculum) since university and that was a long time ago." One teacher questioned "How do you teach a person to think?" Time, and the lack of it, and the shortage of support materials represented another area for discussion. Another common concern expressed was whether students would be able to handle the material assigned to each grade level. Throughout the exercise the level of discomfort of the participants in not knowing all the answers to their concerns was evident.

Despite the initial concerns expressed by the participants, as time passed there were several groups that began to comment on how well organized the protocol really was. One participant commented "I see, the lights are going on." and went on to

praise the language of the outcomes for not only stating what was to be done but also for suggesting contexts which would provide students with connections. Comments such as "you're doing the same teaching but with a different goal in mind" and "so it looks like common is going to try and bring both of the ideas together and then just kind of branch off after that. But like they're two different kinds of thinking" seemed to summarized many of the groups conclusions as they worked through their part of the document.

Technology was an issue that inevitably entered the discussion. This is an area of mathematics education where the range of skills and level of competence varies immensely. In one group the discussion focused around whether or not these courses could be taught without technology. An interesting discussion of the role of technology in the classroom and the availability of technology in all schools and to all students ensued. One participant went so far as to assume the role of a teacher from a small rural school with no access to computer technology and with a large percentage of the school population that could not afford to purchase graphing calculators. He pressed the other members of his group to help him find a resolution to the problem of how he would deal with those aspects of the program that required the use of technology. Their discussion opened many avenues for him to explore. Other discussions focussed on options available for mathematics teachers to improve both their own computer skills and their ability to implement them in the classroom. This overriding concern with the role of technology reappeared in many participants' comments in the final survey.

Interspersed with all of these issues was a significant amount of mathematical discourse about how individual teachers approach mathematics topics and deal with

student difficulties. From the perspective of a researcher watching and listening to these discussions evolve, the group dynamics and the reactions of participants were of particular significance. It seemed in each group that a leader gradually emerged. The role of the leader was to keep the group focused. Members of the group were quick to reassure others in their group that they were not alone in their confusion or frustration. The sharing of ideas and concerns seemed to develop an informal bond between the members of a group, something similar to a club membership. There appeared to be a sense of safety in expressing their concerns and criticisms individually within their small groups. Once they had received collegial support for their concerns the participants felt empowered to share these concerns publicly.

The early frustration of the participants with the document made it very tempting for me to draw the groups back together and walk them through the organization of the document. I cannot count the number of times I have done this in a classroom convinced that my instructions had not been clear enough. This need to keep things moving in the direction you had planned is part of the control issue we as teachers continue to struggle with on an ongoing basis. But, I reminded myself, I had designed the activity for them to explore and become familiar with the parts of the protocol that would serve them best. This regrouping would only have put my perspective on the exploration. I would have been determining the outcome that was right - not allowing them to find solutions.

The informal feedback from the majority of the participants after the class was that they found this a very useful exercise and had learned a considerable amount more than if they had just had to sit and listen. However they had little confidence that they had captured all the important factors and almost unanimously requested a

copy of my strand breakdown. A few participants pointed out that the exercise was a waste of time, since I had already done it, and suggested I should just have given them a copy and gone on to do something more useful. It was not surprising to me that one of these participants did not return to future sessions.

In closing I asked the group to reflect on this activity and its implications for teaching. I knew early in the activity that this reflection would be the natural starting point for the next workshop.

Experiencing Occasions for Mathematical Learning

Intentions

Our initial discussion period, for the second meeting on November 18, 1998, centered around the answers I had been able obtain to questions that had been posed about implementation issues. The assurance that someone was aware of their concerns and that an ongoing dialogue had been established removed these concerns as stumbling blocks to discussion about mathematics and how mathematics learning can and should take place. My objectives for the balance of the second class were two-fold: reflection on the jigsaw activity and getting involved in exploration based mathematics.

Reflections: Being Involved in Active Learning

I wanted teachers to reflect on how they felt about the jigsaw activity as a method of introducing material both for themselves as learners and for their students. In addition, I asked them to consider which teaching techniques would be useful in integrating the processes described in the protocol into their lessons. These small

group discussions were tape recorded and the groups also submitted written comments.

In general there was positive feedback on the jigsaw activity. The groups all agreed that they learned more by being actively involved in the learning process then if they had been passive recipients of the information. They particularly valued the opportunity to get a "big picture" sense of the three grades and to see how things "fit together". They recognized in their discussion that often we lecture to students and inundate them with information because, we believe, it will be the most efficient method of transmitting the information. Members in one group commented that, if they had been given the summary document at the beginning it would have gone on a shelf and probably never would have been looked at and that much of the information would have been filed away as information overload. However, after participating in the exercise and realizing the amount of information there was to absorb, they assigned a much higher value to the document and spent a considerable amount of their own time examining it. Suggestions for improving the activity included: prior work by participants in order for them to become familiar with the document and the vocabulary; a facilitator in each group that could provide leadership in the discussion; an overview of the protocol prior to the activity; more time to complete such a complex task. There was a high degree of consensus that this type of exercise "forced them to learn and to get involved with learning." From my point of view, as the facilitator, this discussion provided valuable input for planning future activities as well as confirmation that the jigsaw activity was a productive approach to collective learning.

Discussions about using this type of activity with students raised some interesting questions. Frequently heard comments included "What happens if students go off on the wrong track or do things wrong?" "How do you plan for this and how do you correct it without interfering in the discussion too much?" "What do you do about students who refuse to participate in the discussion?" This last question received an insightful reply, in one group, when a participant commented "Those are often the same kids who refuse to get involved in any learning." There was general consensus between the groups that the benefits of active learning outweighed the disadvantages. However, they did acknowledge that although they often employed participatory activities with their non-academic classes there were very pressing reasons why they often resorted to more passive learning with their academic classes. Needless to say these reasons included the amount of content to be covered and the time pressures. The choice to return to standard methods was not one made without regret. The following exchange of comments summarizes these feelings succinctly:

> "That's when a person relies on the lecture. You have to catch up on your time line."
>
> "Yah because it may not be the best but it's efficient."
>
> "But certainly I think the students can do what we did with the exercise. You know you looked at something and get the idea and retain it longer as opposed to say if somebody just talked to you and told you what it was."
>
> "The key is understanding, understanding ...? and doing that kind of activity."
>
> "I think it stays with them longer but it takes more time so your in a catch 22."
>
> "Yah so I think it's a case of pick and choose and like I say there are times you're just gonna have to get on the blackboard or overhead and give er."

The discussion of the types of teaching activities that would promote the use of the seven processes outlined in the protocol was inspiring and uplifting. As I moved from group to group and in listening to the tapes it was encouraging to hear teachers sharing how they currently incorporate each of these mathematical processes in their classes. A wealth of ideas were shared. Teachers were given reassurance and compliments from their colleagues that their ideas were sound mathematically and that their procedures reflected good teaching approaches. One particular discussion focussed on a class designed by one teacher for students who had experienced difficulty in previous math courses. In the discussion the teachers focused on what skills these students needed to develop to be successful and some of alternate methods that could be employed to provide these students with an another approach to the topics. The importance these teachers attached to creating connections for students was evident in comments such as " I love when you can show kids how things fit together." As the discussions proceeded participants pulled examples from day to day classes that referred to models, visualization, connecting with other subject areas to find applications, and "real problem solving". The expectations these teachers place on themselves to include all of these aspects of mathematical learning was exemplified by the comment of one teacher "I don't do enough" to which a colleague replied "Nobody does as much as they should. We need help setting real problems for kids to work on."

I found it very difficult to stop this discussion as there are so few occasions where such reflection and learning from other professionals can take place.

Actions: Exploring Mathematical Activities

The second objective of this class was to have each of the teachers participate in a series of mathematical activities that related to specific outcomes in the protocol. During the two years I had been studying at the University I had been collecting performance activities, exploration activities, and unusual tasks as well as scoring rubrics for mathematical assessment. What I now had to do was choose three of these that would represent a range of activities that teachers could readily see themselves using in their classrooms. I wanted to make sure that at least one of the activities dealt with topics that could be used with Math 30 students. One of the critical issues for teachers of academic courses is that activity based learning is time consuming and so I endeavored to find activities that had the potential to provide learning that crossed a wide range of outcomes. In each activity I tried to incorporate some of the tools of mathematics outlined in the protocol and discussed in the NCTM standards. In trying to do all of these things at once, I clearly suffered from the same concern of the teachers and hadn't internalized the lesson from the first class - discussion and reflection take time. Once again, I planned too much and we ended up feeling very rushed. It would have been more beneficial to have spent a longer period of time on two activities and allowed time for reflection on the same night. As it was, the reflection exercise was put off until the next class, which was not until January. This is never as effective and the feedback I received was not as enthusiastic as the teachers' reactions had been on the night of the activities.

An important lesson I learned from doing activities with students in the classroom was that the more open ended the activity is the more important is the role of the teacher as a facilitator and source of information. Students need to be able to

choose how they will respond to the activity presented and they need to have the opportunity to bring their own experiences to the situation but they often need assistance with getting started or in finding their way. Sometimes a carefully asked question can turn confusion into enlightenment. The teacher is often a key component of the environment that helps the student to define the situation. The success of the activity portion of this class is largely due to the expertise and assistance of my two colleagues, Florence Glanfield and Kathy McCabe, both of whom have been actively involved in developing teacher inservices for the protocol. Each of us facilitated one activity and worked through it with groups of teachers. It provided us with an opportunity to watch three groups of teachers explore the same activity and marvel at the different ways there are of arriving at a conclusion. It also reinforced for us the skills we needed to develop as facilitators and the pitfalls that must be avoided to not fall into the role of director as you repeat the same activity. The temptation to shortcut discussion and guide discovery is strong, especially when time is a factor. It becomes increasingly important to discipline yourself to listen to the input of the participants for new ideas and not only for the ones that will lead quickly in a desirable direction. One must constantly keep in mind that the outcomes of the activity should not be narrowly defined - participants must be able to bring their own experiences and meanings to the activity and its outcomes.

The Outcomes of the Volume Activity - Expected and Unexpected

It is easiest to characterize some of the generalizations from this evening by talking about my perceptions of the three groups who worked with me on the Volume Activity (Appendix F). This activity asked participants to verify the formula for the

volume of a sphere by taking measurements of the amount of water displaced by a series of different diameter spheres and then making calculations and plotting their calculations. This is an activity rich in mathematical skills that also requires a significant amount of problem solving on the part of the participants. The first surprise for me was that, in general, the teachers had a very poor sense of volume. They could not estimate the volume of the spheres and several discussions centered around whether the values they were getting were reasonable. As I had done this activity previously with Physics teachers, where this discussion did not arise, it reinforced the importance of starting where students are at and not making assumptions. This was further emphasized as the teachers began to discuss the way in which they would measure the diameter of the sphere. What ensued, in each of the three groups I worked with, were the most interesting problem solving discussions. One group struggled with finding a suitable tool for measuring the circumference of the spheres. Several methods were discussed and discarded as impractical or inaccurate. The method they chose for the larger spheres was not as effective with the smaller spheres. The discussion soon became one of various tools that could be made available for measurement and the impact this would have on the outcomes of the activity. It would have been a much more efficient activity if they had been provided with a method for measuring diameter, but the mathematical discourse that preceded their solutions to this problem, which was a valuable experience for all, would have been lost. In the end, each group arrived at a slightly different method of measurement and all groups spent some time discussing measurement error. Although this was not an expected outcome of the activity, it is in fact a specific outcome of the shape and space strand of the protocol.

Another observation that proved interesting was that each group that worked through the exercise chose to approach the graphing component differently. One group chose to use their graphing calculators to do the graphical analysis. The discussion that followed in this group showed a wide range of levels of competence and comfort with graphing calculators and a very worthwhile discussion concerning the usefulness of graphing calculators in the classroom and the relative properties of different calculators resulted. The second group resisted drawing the graph as suggested in the instructions and tried to calculate the constant in the equation using the raw data. This group felt that the graphing exercise was unnecessary. This is an interesting perspective from a group of teachers who teach graphing as a useful mathematical tool. The third group followed the suggested instructions and not surprisingly were done more quickly than the other groups. Their discussion centered more around difficulties of doing this exercise with students and not the mathematics of the problem. It is always interesting to see the different ways in which group dynamics develop and impact the way a group works together. Since we rotated the groups through the three activities it may well be that the order in which they did the activities may have had an impact on their reactions. In addition as the teachers became more comfortable with other group members more discussion was evident.

Two additional discussions that occurred during this exercise are worth sharing as they represent key issues that mathematics educators need to address if we are to successfully move towards more open-ended activities in mathematics. The first discussion centered around when and how one would introduce this activity to a class of students. Would it serve as a unit introduction? A year end review? A form of cumulative test? This discussion initially arose because there were so many

outcomes covered in this one activity. The initial discussion seemed to suggest that the students would need all of the skills required before they could begin the activity. Therefore these teachers felt the activity could only be used as a review or a test. But other members of the group argued that would defeat the attempt to introduce a discovery approach to the mathematics we were teaching and that students should be introduced to the skills as they needed them to do the task. Thus, in their minds the activity presented teaching opportunities. The debate that followed often returned to the issue of time and the perspective that many teachers share and have to overcome - a point of view that sees these open-ended activities serving as motivational add-ons to our regular instruction. We need to move away from the belief that in order to reinforce what students "discover" we need to "teach it" as well. This was one of the topics that emerged from these activities that I used as a starting point in the next class. I asked teachers to focus on what kinds of outcomes can best be introduced with open-ended activities and which ones are more suited to other teaching techniques. I asked participants to consider what changes they feel must occur if we are to start rethinking what it means to teach mathematics and to change our daily classroom procedures.

The second discussion that clearly revealed existing attitudes that must be addressed in the attempt to change the way we approach mathematics teaching was introduced by one of the group members who teaches both Math and Sciences. Her enthusiastic comment "I can hardly wait to go back and try this activity with my Science 10 students!" exemplifies how we associate this "lab" type of activity with subject areas other than mathematics. When asked why she felt it would be better to do this with her science students than her math students she commented that the

activity provided an opportunity for students to practice a wide variety of skills that were specific outcomes of the Science curriculum and she could see exactly how it would fit. This curriculum fit was not as immediately obvious for any of her Math classes. She saw the activities as coming from many different units and courses and this seemed problematic. In reality, the skills in the Science course also come from many different units, but lab work is often designed to have students draw on skills from several areas of science. A secondary issue that arose from this discussion centered around classroom management of lab activities and this we decided was too important a discussion to keep within the small group. I agreed to put this on the agenda for the next class.

Creating Occasions for Mathematical Learning

Intentions

My original intention for the third meeting of this group was to provide the majority of the time for the participants to work with a colleague to develop an activity, based on the protocol, that they could try with their students and report back to the group in the final workshop. As I had already discovered, much of what was to transpire in each class was largely determined by the last class or events that happened in the intervening weeks and months and the directions that seemed most useful for participants. Our third class meeting, January 23, 1998, coincided with the announcement by Alberta Education of a delay in the mandatory implementation of the Applied Program until September 1999. Many of the participants, in addition to the input they had given me to share with the Curriculum Branch staff, had been

actively lobbying personally and through their school boards for a delay in the implementation of the Pure and Applied mathematics programs. They felt that resources were not available and that the mathematics community had not had sufficient opportunity to prepare for such a significant change. A lengthy discussion concerning the implications of this decision was inevitable and many participants used this time as an opportunity to gather information from others about possible approaches to addressing the split implementation dates. Teachers expressed a sense of empowerment as a result of their role in this decision. Although the delay was not all they had hoped for it represented an acknowledgement, to them, that their concerns were seen as real and valid. A secondary discussion about the implications of the new programs on assessment and the Calculator Policy provided me with valuable input into discussions that were ongoing at Student Evaluation. The focus on technology in the new program will lead to significant changes to how student performance is assessed and to the type of technology that will be required for mathematics exams in the future.

Reflections: The Exploration Activities

In next portion of the class I asked groups to reflect on the activities from the last workshop and to complete a questionnaire that focused on how the activities related to the strands, processes and specific learning outcomes of the protocol as well as how they related to the NCTM Professional Teaching Standards (1989). Once this was completed they were to begin a discussion about what kinds of activities they would design. I encouraged them to think about the following questions that I had gleaned from their discussions in the previous class:

• What benefits would students gain from these activities?

• What new outcomes in the protocol do not lend themselves to traditional teaching methods?

• How do you focus students on what they have learned?

• Are there outcomes in different strands that are connected that might be incorporated into one activity?

• What would they do to teach intended outcomes that didn't emerge from the discovery lessons?

In their feedback on the activities from the last class the groups indicated that the activities had been relevant, interesting, and clearly related to the protocol. They commented that the three activities chosen had addressed a broad range of specific outcomes and that the processes outlined in the protocol philosophy were well represented. A similar conclusion was drawn with regard to the NCTM professional teaching standards. There were several groups that commented on the value they saw for themselves in the opportunity to communicate with others their mathematical ideas and felt this should be a focus of all activities. Comments regarding activities that required a lab approach included expressions of concern about time for lab set up, management of lab activities in a traditional classroom, availability of materials and the need to guide discovery carefully. These comments represented an insight into the true position of many mathematics teachers (one referred to in the literature as trivial constructivist) on the continuum from traditional approaches to teaching in a more open and enactivist way. The issue of the teacher maintaining control of the learning environment and the learning outcomes was very evident in these discussions. There

was widespread agreement among the group that this type of activity based learning is valuable for students and teachers. However, this enthusiasm was tempered by the realities of day to day demands and evaluation processes which continued to cause concerns. Many of the comments that emerged from this discussion were generic in nature and not directly related to the activities from last class. I felt this was because too much time had elapsed between the activities and the reflection process to obtain the most valid feedback.

I had not forgotten my promise to bring up the discussion of management of lab based activities but we agreed because of time to delay this discussion until the end of class. Two of the teachers who taught Science courses as well as Math volunteered to begin the discussion with some suggestions based on techniques used in their Science classes.

Actions: Creating Occasions for Mathematical Learning

At this point the group was asked to work in pairs on creating an activity that would reflect specific learning outcomes from either Math 10 Pure or Math 10 Applied. The intention was that once the activity was designed they were to take it back to the classroom, try it out with their students and share the activity and their experiences in the final workshop.

The discussions I had with individual groups as they worked on this assignment reflected the wealth of experience and imagination present in this collection of teachers. I spent time talking with one pair of teachers about creating a list of contextual starters for mathematical learning. This focus on relating mathematics to everyday life is a difficult one for many teachers as they do not

necessarily see the mathematics around them. The high school science programs have had a Science, Technology and Society emphasis for the past several years and many science teachers have developed the ability to see applications at every turn in the road. Although it is not as easy to see mathematical applications it is a skill we as mathematics teachers will have to work on developing. I encouraged them to proceed as I felt, as they did, that this would be a valuable tool. Another group who took a similar perspective talked with me about creating a series of activities designed around one strand and based on a current theme. Their idea was to relate as much of the measurement strand as possible to the upcoming Olympics. The brainstorming we did gave them a good start and served to further emphasize the importance of teachers having the time and the opportunity to share and discuss their ideas.

Another group was working on a spreadsheet exercise for students and when I inquired how things were going one of them commented to me "there is a lot of learning going on here." As they experimented with inputting data into the spreadsheet they began to realize some of the skills that students would have to learn to work effectively with this tool. My own recent experimentation with copy and repeat commands helped them with their problems and again this illustrated how important hands-on experience, as learners, is for teachers. Their recent experiences with learning the required skills make it much easier for them to guide students in their learning.

In different groups the discussions focused around a multitude of topics and a wide range of approaches to the task emerged. These included: designing activities that covered several outcomes in such a way that some tasks lead to reinforcement and others led to discovery; exploring the new texts to see what kind of activities

were suggested there and modifying them to suit particular situations; researching materials in the library and on the internet; working through problems to get a handle on mathematical concepts that were new to the course. The various approaches to this task gave further insight into the strengths and weaknesses of individuals in the class. It was becoming increasingly clear where participants felt comfortable and in which areas they felt less secure. By encouraging people to work with a colleague with a different approach than they traditionally used I hoped to encourage some sharing of experiences and some risk taking. I firmly believed that the most effective learning that would take place would be between the participants and that my role was to provide opportunities for this learning to occur.

Discussion of Lab Based Activities

When we reconvened for the last half-hour I asked the two volunteers to lead a discussion on classroom management of lab based activities. The discussion itself was interesting and dealt mainly with management issues. For me the most interesting aspect of the discussion was the level of discomfort and concern participants expressed regarding the loss of control of the learning outcomes. This is a significant philosophical issue for mathematics teachers and one that will need to be addressed head-on in inservices designed to help teachers rethink the methods we use in teaching mathematics.

Another related concern for teachers that emerged involved issues connected to evaluation - in particular group evaluation. There is a definite atmosphere of accountability in the school setting in Alberta and this concern was never far from the minds of the participants. Their concerns about how open-ended discovery and

performance assessments can become part of a student's formal evaluation are shared by the members of the Student Evaluation Branch. In order for these types of learning to take hold in the classroom they will have to form some part of the students final evaluation. The Alberta Education *Classroom Assessment Materials* have attempted to introduce this idea to teachers. However, until there is a widespread belief that both Achievement and Diploma examinations will test these outcomes alternate approaches to teaching and assessments will not be used in many classrooms. The role of the graphing calculator and computers on assessment also played a central role in this discussion. Participants debated the pros and cons of extensive use of these tools and their appropriate place in the overall picture of student learning.

Sharing Mathematical Creations and Reflecting on Student Responses

Intentions

The final workshop was designed to be an opportunity for participants to share the activity they had developed in the previous class and to reflect with others on how successful it had been in their classroom. It was not without a great deal of personal concern that I left the agenda for this last class open to provide ample time for sharing and discussion. In fact, the truth of the matter is that, in order to satisfy my own needs to maintain control of the situation, I had a series of back-up plans and activities just in case the participants had not done their "homework". I had worried unnecessarily and the planned 5 - 10 minute presentations from each pair of teachers indeed took longer than planned. However, the presentations provided the entire group with a wealth of ideas and raised important questions.

Actions and Reflections: Reporting on Mathematical Occasions in the Classroom

Each group was asked in their presentation to provide a handout for the others listing the strand, general and specific outcomes and processes for their task. They were also to include a handout or list of instructions to students and tips for successful implementation for teachers. Although each task presented led to an interesting discussion I will focus on some of the more general observations.

Outcomes that were addressed crossed all of the strands and were drawn from the pure, the applied and the common outcomes contained in the program of studies. No one area of mathematics proved to be impossible to approach using one of the processes outlined in the protocol philosophy. Technology played a role in many, but not all, of the applications and in many cases the sophistication of the technology and its application were not complex. The role of communicating mathematical ideas was central to many of the activities, as was the importance of providing an activity that had a real life contextual base. There was a conscious attempt by all presenters to find an activity that made learning math at least interesting if not necessarily fun. Group work played an integral role in many of the activities and it was mentioned on a number of occasions that not only was mathematics being taught here but also some social skills. Many of the activities, although somewhat open-ended in their directions and with an element of problem solving involved, were clearly designed to reinforce existing skills and not designed for creating opportunities for new mathematical learning. Some activities began with a significant amount of traditional teaching and in these cases there was no apology for this. Some of the participants

did not design new activities but used variations of existing activities and adjusted them to meet the outcomes of the protocol.

Each of these observations combined with the discussion that accompanied the presentations lead me to believe that for many teachers the move to a more open classroom had already begun. These teachers exhibited a great deal of skill in describing activities that allowed the students to develop their skills, starting from where they currently were in their problem solving. The designers recognized the need to use technology, where applicable, as a tool to facilitate mathematical learning, while continually reminding students of the need to estimate and check that answers were realistic based on the context of the question. They presented problems based on contexts that required students to make connections from outside the mathematics classroom and to bring this information to their problem solving strategies. The importance of working cooperatively and communicating clearly was given a prominent place in their activities. The design of many of the activities took into account the need for students to draw on the skills of listening and visualization in working toward a solution.

From the discussions that took place it was clear that these teachers valued the opportunity not only to share their knowledge and experiences but to learn from those of others. They used the workshops as an opportunity to assure themselves that their doubts and concerns about everyday issues were not held alone. One teacher asked of one presenter, "Do you really expect kids will ask questions?"; another asked "How do you deal with the logistics of the noise in the hallways?"; and yet another commented "High school kids aren't afraid of manipulatives and they don't feel silly using them, but we as teachers do. Whose problem is that?" The teachers also took

advantage of the discussion time to share ideas about resources that were useful and to explore how the activities could be enriched. On several occasions teachers commented that after working on their activity they encountered something in their daily life that "fit" with their project and could be used to improve it.

This final workshop served to convince me that, at least for that particular day, many of these teachers could see potential for changing the way we approach the teaching of mathematics to our students, at least on a partial basis. This is not to say there were not skeptics in the crowd and you could easily identify them from the types of questions they were asking. Their questions revealed concerns about the management issues and whether these tasks would in fact lead to "significant learning" - learning that was required by the curriculum and that would be tested. But, I believe, these questions were not asked to be difficult nor to put down others, but out of a true concern for ensuring that students continue to learn what they need to know to be successful within the existing system. The mind set that must change, if a new approach to mathematics learning is to emerge, is that we as teachers know what it is that students need to know and how best to make sure they learn it.

The Post-Session Survey

The final survey was designed to be completed at the end of this last workshop but as usual we were out of time and energy. I asked participants to complete the survey on their own time and return it to me by mail or by fax. The timing of this was not good as it was the beginning of spring break and so the survey results came in slowly and many were not returned. I did have the opportunity to have informal discussions with many of the participants after the completion of the course during

marking sessions and during the summer months and these comments along with the final survey results will be included in the next chapter in the summary of my findings. The responses that were returned provided valuable insights and information. A summary of the compiled responses is included in Appendix E.

Chapter 4 Summary of Conclusions and Recommendations

At this point it seems appropriate to reflect on the original research question and how well I have answered the questions I posed in the beginning of this project. Did I succeed in **creating occasions that encouraged teachers to reflect on and make changes to their pedagogical content knowledge**? What lessons can be shared with other professional development providers based on this experience? What views did participants have with respect to teaching and change when they entered the workshop and did the experiences of the workshops have any impact on these views? What factors served as inhibitors to change? Did the enactivist perspective prove to be a useful model for providing teacher inservice? And finally, probably the most important question that needs to be asked at the end of a undertaking like this is - What next? What concrete steps can be taken to ensure that these teachers and others become reflective practitioners?

The Participants' Perspectives

The participants in this project represented a cross-section of mathematics teachers who are committed to quality education and to making the changes prescribed in the programs work for students. They bring to the inservice a wealth of knowledge that goes beyond their mathematical content knowledge and extends to include knowledge of the students they teach, assessment strategies, methods of instruction and the idiosyncrasies of their teaching situation. In addition they bring their own pedagogical content knowledge - the knowledge they possess of how these different types of knowledge interact with each other and how they can be used to

supplement each other in working toward effective teaching (Schulman, 1986, as cited in Loucks-Horsley, Hewson, Love & Stiles,1998, p.36).

I think it is important to point out, before making any generalizations, that collectively this group of mathematics teachers may be much farther along in the process of accepting the NCTM Standards and the philosophy of the Western Canadian Protocol as the future direction for mathematics education than the average classroom teacher of mathematics. Their willingness to become involved in this project almost a year before the implementation date and their concerns about many issues indicate a level of awareness and concern that may only be found among leaders in the mathematics community. Although many mathematics teachers may share their beliefs about mathematics teaching, the level of commitment to change and the recognition that change is required if we are to improve students mathematical thinking, may not be as high in the general population.

The participants in this group indicated clearly that they believed that students benefit most from mathematics instruction that can be related to their daily lives. They believe that student centered and student directed learning are the key components of successful programs. They recognize that all students can learn mathematics and that different students learn in different ways. The discussions they had clearly indicated that they valued the role of communication and group work as important tools in mathematical learning. These teachers had already begun to internalize the philosophy of the protocol, incorporate mathematical processes into their teaching and had moved a long way towards the goal of mathematics education that enables students to become critical thinkers and creative problem solvers.

As stated earlier, the change cycle, as outline by Posner, Strike, Hewson, & Gertzog (1982), consists of four stages:

1. Dissatisfaction with the existing concept.

2. An understanding of why the new paradigm makes sense.

3. A belief that the new conception is plausible.

4. The view that the new concept has the potential to lead to new knowledge.

If I were to reflect on the position of these teachers with respect to this cycle, I would argue that they entered the inservice sessions at stage two. They were not satisfied that the current curriculum and were not certain that the generally accepted methods of presenting it were meeting the needs of all students. The earlier program revisions and the NCTM standards had offered starting points for change and the new proposed curriculum changes promised to move mathematics education farther towards meeting these needs. But the teachers were uncertain if the proposed change would solve problems or only create new ones. They needed to convince themselves that the proposed changes would lead to better learning for students. They were already experimenting in their classes with hands on activities and open ended problem solving. Mathematics teaching in their classrooms had begun to move towards involving the whole student in collaborative and interdisciplinary tasks and encouraging risk taking. The concern that existed for them was the uncertainty as to whether what they were doing was successful and so, to ensure the best advantage for their students, they tended to return to tried and true methods of instructional delivery to reinforce activity based learning. These teachers needed an opportunity to move to the third stage of the cycle where they could view the changes proposed as plausible, desirable and manageable. They needed to be given the opportunity to determine if

the proposed changes offered gateways to enhanced learning. As the comments from the final survey indicated, by the end of the sessions many of the participants had moved to stage four of the cycle where they had concluded that the proposed changes offered positive new directions to explore and opportunities to extend students' knowledge of mathematics. The extent to which these intentions to make changes will be followed with concrete actions, as the new programs are implemented this fall, will to a large extent depend on individual teaching situation. The nature of the resources available, the availability of collegial support and ongoing opportunities for sharing and reflection will have a significant impact on promoting ongoing change.

Although the teachers' responses to the initial survey indicated some discrepancy between what they believed mathematics education should be and what they said they were doing in the classroom, subsequent discussions would seem to contradict this. Much open-ended problem solving and student directed active learning appears to be taking place in their classrooms. The reasons for this divergence, between what they believe should be done and what is actually happening in their classrooms, are clearly recorded in their responses to the survey questions and in their deliberations. These teachers feel pulled in two directions by the same master. From their viewpoint they are faced with a conundrum over which they have no control. On the one hand the provincial government continues to operate in a climate of accountability and restraint where marks on provincial exams are viewed as a measure of success and funding for education is tightly controlled. This attitude, compounded by the content demands of the curriculum, does not leave teachers time to prepare for and experiment with performance assessments and other open-ended activities in their classrooms to the extent they feel is adequate. They feel the dollars

available in school budgets for computer and calculator technology will only be obtained by "doing battle" with other departments for limited resources. Since mathematics in many schools has always been viewed as an "inexpensive" subject to teach, this shift in resources will be hard to achieve. On the other hand they see the new program, mandated by a different branch of the same government, advocating a more active approach to mathematics education - one that requires rethinking how we approach mathematics and requires extensive use of technology. Combining these perceived contradictions with the lack of information about implementation details and resources created an atmosphere of frustration that presented a serious impediment to real change. It was only by setting aside time in each session to deal with these practical and pressing issues and, thereby, removing them as obstacles to change that we were able to then focus on different methods for the teaching and learning of mathematics.

The Enactivist Perspective and Action Research as an Model for Teacher Inservice

The choice of action research as a research design allowed me to weave the themes of this form of research with the tenets of enactivism. Empowerment of participants, collaboration through participation, acquisition of knowledge, and social change, the four basic themes of action research, were all outcomes that were achieved through the course of these sessions. The importance of this form of research in the educational field is clearly outlined in the comments of the editor of *Educational Researcher.*

> "Our growing awareness of the complexity of the issues related to education makes it more important than ever that we employ the skills of the social

sciences in our educational decision making. I believe it is time to begin to talk with each other, to listen carefully to what others - including those who questions about the work each of us does" (Donmoyer, 1996).

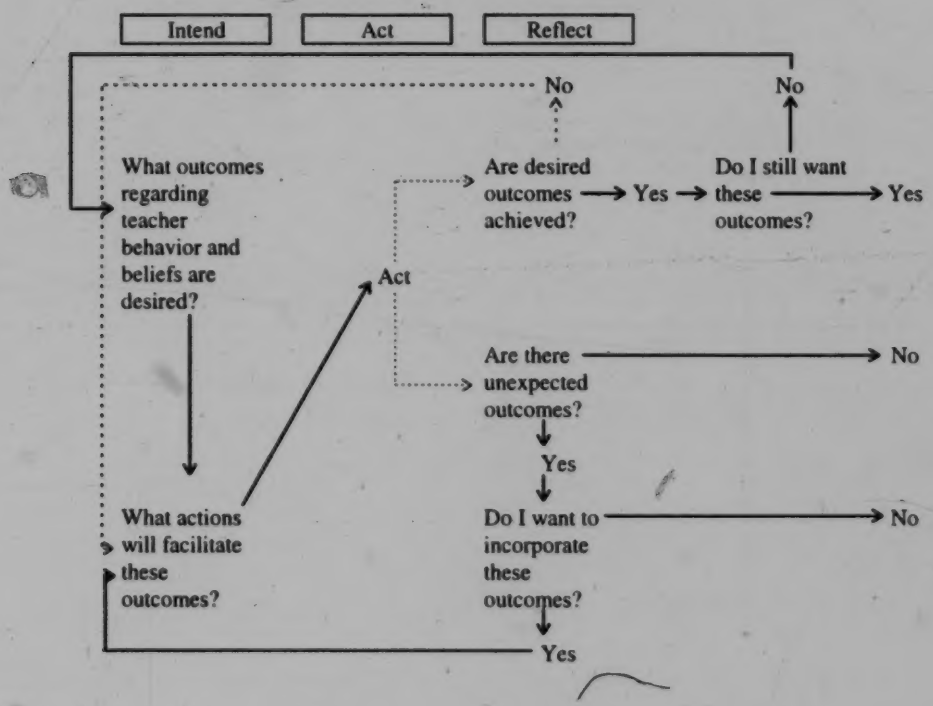
I used these themes and comments as constant reminders as I worked through the planning and implementing of each session.

The goals of the workshops included creating an atmosphere where mathematical discourse and conversation could occur freely. In keeping with the enactivist perspective my place as teacher was to be "In the Middle" occasioning learning and listening to the individual's perceptions of reality. This differed from more traditional approaches where the teacher works "over the shoulder" guiding instruction or "in front" of the students telling them the truths of mathematics and teaching. Being consciously placed in this role as a group facilitator helped me to gain insight into some of the problems faced by teachers trying to implement these strategies in the classroom. The temptation to construct reality for participants and to maintain control of the direction and outcomes of the learning situation was as real and as difficult for me, in this setting, as it is for teachers in the classroom. But the advantage of allowing the structure of each individual participants' values and experiences to contribute to the direction the sessions would take, is evident in their degree of satisfaction with the sessions, as exemplified in their comments and their discussions. Each of the components of the workshops is singled out by one or more of the participants as being particularly useful and the general themes of communication and reflection are evident throughout their comments.

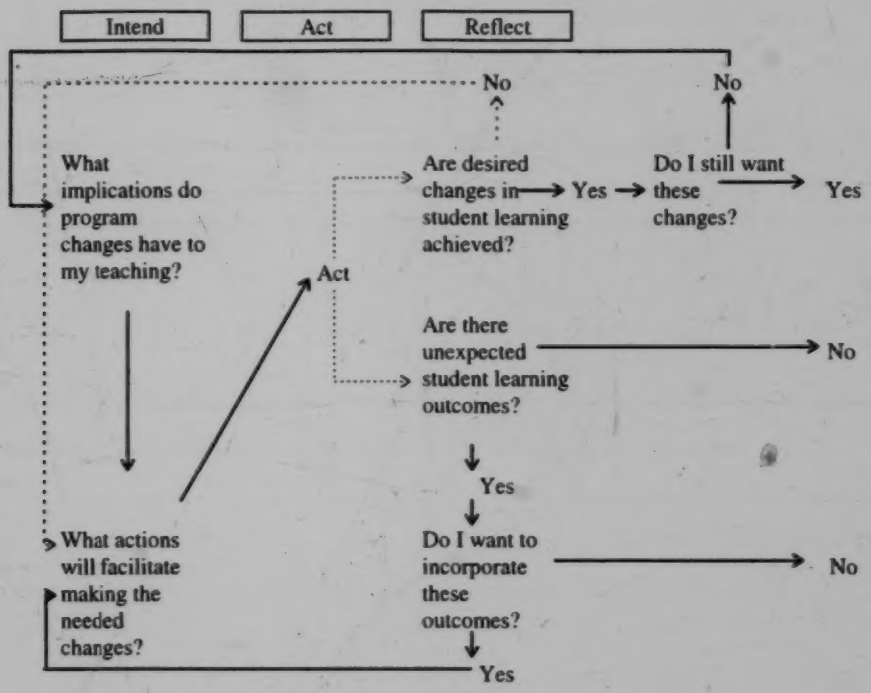
An important feature for me, and therefore for others that approach inservice in a similar manner, as well as for teachers employing an enactivist perspective in

their classrooms, is that as the facilitator this was an ongoing learning experience. I, similar to the participants in the workshops, was embarking on an action research project designed to examine my teaching approaches. In many ways the dual nature of the action research served to enrich the experience and emphasize the importance of each stage of the research cycle. As I reflected on the parallels between my learning and that of the workshop participants, the research cycle became a reference point. Each intention, each action and each reflection, must be viewed from a personal as well as a group perspective. Each individual's previous learning and experiences contributed to the sense they made from the learning experiences and helped to define the future directions they choose. The adaptations of Dick's research cycle shown below attempts to capture these parallels.

The Facilitator's Research Cycle (adapted from Dick, 1998)



The Participant's Research Cycle (adapted from Dick, 1998)



At each stage of the process it was important from both perspectives to examine motivations, question procedures, take action based on previous outcomes and reflect on the entire cycle. From my perspective the outcomes were never completely predetermined and the directions we would move often were not those that I had planned. To be flexible and responsive to the needs of the group is a skill we, as mathematics teachers and professional development providers, will need to develop with time and practice. We will learn to appreciate learning with our students and looking to them for new directions as opposed to lecturing to our students and listening for the right answers. In the later sessions, teaching became learning and was viewed as a reciprocal activity where all participants learned

together. The knowledge that was gained was not disembodied knowledge but knowledge that came with a personal connection. The way in which these teachers perceived their role in the change process was reflected in the actions they chose. Their actions were perceptually guided in a true enactivist sense and the workshops served to provide a vehicle for structural change and a foothold for new ideas.

In reviewing the workshops and their design I found myself reflecting on how the characteristics of a successful professional development plan are embedded in this type of approach. In his summary of the ten key principals for successful inservice, Doug Clarke (1994), emphasizes the need for the inservice to be open to suggestions from participants and to address issues of concern and interest to the participants. The importance of creating participant ownership of the decision making process is also identified as a key component for success. By allowing the participants to determine, through their discussions, the direction of future workshops, meeting these criteria was ensured. Another aspect of successful inservice is described as recognizing impediments to change and recognizing that change is itself a slow and painful process. Our discussions at the beginning of each session, regarding implementation issues, served to meet this need for most participants and to provide them with opportunities for ongoing support and input into problem solving from peers. Encouraging teachers to share and reflect on their personal classroom experiences provided validation that the suggested outcomes of these sessions could provide positive results. The time intervals between sessions were intentionally set up to provide teachers with ample time to experiment and reflect on the outcomes of the previous session. This also served to establish an ongoing support system which was a key component in developing group cohesiveness and a sense of togetherness.

The camaraderie and network developed during these sessions will continue to serve many of these teachers throughout the implementation years.

In examining the data collected from this experience six key components for providing inservice activities to mathematics teachers stand out. These must be kept in the forefront of the professional development provider's plans if the program is to be successful in promoting lasting change. These include:

1. Be flexible. The inservice must meet the needs of the participants and must consider both their current situation and the pedagogical content knowledge they have acquired over time. The inservice provider must recognize early in the workshops where the participants are in the change cycle and plan accordingly.
2. Be sensitive to current issues and concerns. If these are not identified and addressed they will serve as blockers to change and progress. Although all impediments to change cannot be removed, acknowledging them provides a better learning environment.
3. Actively involve yourself and the participants in doing and learning mathematics. Do not allow yourself or the participant to become passive observers.
4. Provide numerous opportunities for discussion and reflection by the participants and the facilitators. This should include large and small group discussions as well as personal reflections in the form of written comments. This aspect is critical to making sense of the experience and internalizing the lessons learned by all.

5. Spread the experience over time. Allow a network to form that can serve to reinforce and support the changes desired. Provide the participants with time to experiment with and reflect on what they have learned and an opportunity to share their reflections.
6. Use time as an ally not an enemy. Do not try and accomplish too much in any one workshop. Recognize that encouraging mathematical discourse and active listening to colleagues requires time. Many of the most valuable lessons will be learned from these discussions.

Future Directions and Challenges

It is evident from the success of this series of workshops and the popularity of others offered in June, 1998 that mathematics teachers are eager to get involved in and connected to the new curriculum. Inservice that uses the enactivist perspective as a model for learning seems to be an effective tool in developing a positive attitude towards change. Discussions with teachers would indicate that they seem relatively confident that the changes in the Pure Math 10 program are manageable but have major misgivings about their level of preparedness for the Applied Math 10 program. Despite the fact that mandatory implementation of the Applied Program is not until September 1999 over 10,000 students will be entering into Applied Programs this fall. The feedback from the participants, as well as feedback I have received from numerous other sources, suggests without adequate inservice to provide teachers with the opportunities to explore and experiment with the Applied Program it will not be implemented in the true spirit of the protocol. The issue of whose responsibility it is

to provide this inservice is a moot point. The reality of the situation is that without support and occasions to share philosophical and practical perspectives many teachers will feel isolated and alienated from the new curriculum and significant change will not result. Whether this initiative is undertaken by Alberta Education, the publishers of resource materials, the calculator manufacturers, the Mathematics Council of the Alberta Teachers Association, the post-secondary institutions or some combination of these in a cooperative effort is immaterial. Teachers must not feel they have been left to implement changes of this magnitude in isolation.

Inservice for teachers must not be limited to the philosophical aspects of the curriculum. Participants in the workshops clearly indicated that there are areas of mathematical knowledge where they need additional background information. They want and need to have an opportunity to learn the mathematics involved in these outcomes. Another area where knowledge is lacking for many teachers is in the integration and use of graphing calculators and computers in their lessons. The use of spreadsheets as a mathematical tool will require learning new skills for many teachers as well as students. Without an opportunity to delve into these areas as learners teachers will experience difficulty in conveying the true scope to students.

The introduction of the Pure and Applied Programs with their emphasis on problem solving and the extensive use of technology as a mathematical tool will present new and exciting challenges to teachers developing assessment strategies for these courses. The recently released calculator policy, for use with the new programs, will force mathematical educators to reexamine how we approach many traditional tasks such as algebraic manipulation and statistical analysis. Collaborative work with groups of teachers to collectively struggle with these issues and set provincial

standards for the new courses will greatly assist teachers in their attempts to implement change.

The post secondary institutions face a challenge in their field experience programs. To adequately prepare student teachers to adapt to these new approaches curriculum and instruction courses must offer students occasions to experiment with active learning and teaching strategies. Many of these students were highly successful mathematics students in traditional classrooms and unless they venture out to do their student teaching with the confidence to try new approaches they will quickly return to methods that they found worked for them as students.

Opportunities that encourage teacher input into the decision making process, as we move thorough the implementation stages of the new programs, will further contribute to alleviating the unhappiness and sense of frustration many teachers feel at this point in time. I would like to think that, given the opportunity to be actively involved in the development of their own approaches to these programs through interaction with colleagues, teachers would come to view the programs outlined in the Western Canadian Protocol as a positive move in mathematics education. Teachers must grab each chance for collective sharing of ideas and the workload.

This study proved to be a valuable learning experience for me. The lessons I have learned from and with these teachers will have a significant impact on the way in which I approach my own mathematics classroom and future teacher workshops. In retrospect there should be little difference in designing inservice opportunities for mathematics teachers and in planning lessons for mathematics students. In both cases we are trying to establish occasions for learning. In trying to bring closure to this

most enjoyable undertaking and reflecting on the comments of participants I arrived at the following analogy.

Imagine mathematics teaching and learning as a trip to a familiar destination. There are many ways of arriving at the destination but the traditional approach is one where the driver has total say as to which route will be taken. The driver is prepared and willing to do most of the work and the passengers need not get actively involved in the trip. It appears clear, based on experience, that the driver knows the most efficient and best way to get there and can ensure no one will get lost along the way.

Imagine now a new approach to arriving at the same destination. In this new perspective each passenger along with the driver is involved in the planning of the route, what side trips will be taken and shares ownership in the decision making process. It may take longer to get where we are going and we may get lost along the way but in the end we will arrive at the destination and will be wiser for the experience.

I hope we will be able to look back at the changes that occurred in mathematics education for teachers and students in the 1990s and say the changes have made us all richer mathematicians for the experiences we have had.

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Appendices

Appendix A: Implementation Schedule Western Canadian Protocol - Alberta

Year	Applied (RFP Interm materials) Optional Year	Applied (RFP Final materials) Mandatory	Pure (Final materials selected from publishers' submissions)
1998	10		10
1999	20	10	20
2000	30	20	30
2001		30	

Appendix B: Pre-Workshop Survey

Survey

Thank you for taking the time to complete this survey. Your input is valuable and appreciated.
The survey is designed to collect data on teacher beliefs on the nature of mathematics teaching and mathematical learning and how that is reflected in or enhanced by classroom practice.
It consists of 4 sections each containing 5 questions.

SECTION 1 Background Information

1. List the Mathematics courses you are teaching this school year.

   _____
   _____
   _____
   _____

2. Indicate the level of familiarity you have with the underlying philosophy of the Western Canadian Protocol for Mathematics.
   __ I am well aware of the philosophy.
   __ I am aware of the philosophy but am uncertain of details.
   __ I am not familiar with the philosophy.

3. Indicate the level of familiarity you have with the details of the general and specific learning outcomes of the Western Canadian Protocol for the grades you teach.
   __ I am aware of the general and specific learning outcomes for the grades I teach.
   __ I would feel comfortable explaining the general and specific learning outcomes to a colleague.
   __ I am not aware of the general and specific learning outcomes for the grades I teach.

4. Indicate the level of familiarity you have with the NCTM standards on Curriculum and Evaluation.
   __ I am well aware of the standards.
   __ I am aware of the standards but am uncertain of details.
   __ I am not familiar with the standards.

5. List the professional development activities that you have been involved in the last two years that have been useful to you and indicate what it was about the nature of these activities that made them useful.

   _____
   _____

SECTION 2 Nature of Mathematical Learning

Circle the most appropriate choice.

1. Students learn best when they study mathematics that they can relate to a personal or social application.

Strongly Disagree		Neither Agree nor Disagree		Strongly Agree
1	2	3	4	5

2. Almost all students can learn to think mathematically.

Strongly Disagree		Neither Agree nor Disagree		Strongly Agree
1	2	3	4	5

3. Problem solving should be the main emphasis of mathematics instruction.

Strongly Disagree		Neither Agree nor Disagree		Strongly Agree
1	2	3	4	5

4. It is important to master the basics before learning to solve problems.

Strongly Disagree		Neither Agree nor Disagree		Strongly Agree
1	2	3	4	5

5. Indicate which of the following statements best describes the nature of mathematics from your perspective.

__ Mathematics is a process of inquiry where patterns are generated and then added to the overall knowledge base. Mathematics is not a finished product

__ Mathematics is a static, unified body of truths bound together by logic. Mathematics is therefore discovered not created.

__ Mathematics is a defined set of rules and facts to be used by skilled craftsmen to accomplish a task. Mathematics is absolute and certain.

Comments:

_____

_____

_____

_____

_____

_____

SECTION 3 Teaching Strategies

1. Graphing calculators and computers are important tools in teaching problem solving.

Strongly Disagree		Neither Agree nor Disagree		Strongly Agree
1	2	3	4	5

2. Children can develop problem solving skill by working together in small groups.

Strongly Disagree		Neither Agree nor Disagree		Strongly Agree
1	2	3	4	5

3. Activity based experiences aren't worth the time and expense for what students learn from them.

Strongly Disagree		Neither Agree nor Disagree		Strongly Agree
1	2	3	4	5

4. Students should have the opportunity to write about mathematical topics and their mathematical experiences.

Strongly Disagree		Neither Agree nor Disagree		Strongly Agree
1	2	3	4	5

5. Based on your plans for the current semester estimate the amount of emphasis you place on each of the following activities. You may choose to do this for an individual class or all classes as a collective. If you choose a particular class indicate the kind of class. (i.e. Math 13, Math 9) and if you choose collective indicate the grade levels.

CLASS _____ COLLECTIVE (grade levels) _____

	No Emphasis	Minimal Emphasis	Significant Emphasis
Lecture			
Discussion			
Student reports/projects			
Research (library or internet)			
Students using graphing tools			
Students using computers			
Guest speakers			

	No Emphasis	Minimal Emphasis	Significant Emphasis
Field trips			
Small group work			
Students using manipulatives			
Discussion of homework			
Seatwork			
Tests/quizzes			
Performance Assessments			
Students at the board			
Other Activities (specify)			

_____

_____

SECTION 4 Professional Growth

1. I feel supported by colleagues to try out new ideas in teaching mathematics.

Strongly Disagree		Neither Agree nor Disagree		Strongly Agree
1	2	3	4	5

2. Mathematics teachers in my school meet regularly (formally or informally) to share ideas and materials.

Strongly Disagree		Neither Agree nor Disagree		Strongly Agree
1	2	3	4	5

3. I have time during the school week to work with my peers on mathematics curriculum and instruction.

Strongly Disagree		Neither Agree nor Disagree		Strongly Agree
1	2	3	4	5

4. It is more important to receive continued training in mathematics than it is to receive training in effective ways to teach and manage mathematics classrooms.

Strongly Disagree		Neither Agree nor Disagree		Strongly Agree
1	2	3	4	5

5. I regularly read journals and publications about Mathematics and Mathematics teaching.

Strongly Disagree		Neither Agree nor Disagree		Strongly Agree
1	2	3	4	5

Thank you for you participation. Please feel free to provide any comments.

Appendix C: Post-Workshop Survey

Survey

1. Describe the philosophy and objectives of the Western Canadian Protocol as you understand it.

2. Describe the characteristics of a mathematics classroom you envision based on your understanding of the new APPLIED curriculum.

3. Describe the characteristics of a mathematics classroom you envision based on your understanding of the new Pure curriculum.

4. Which of the activities that we did in this series of workshops were most useful to you? Why?

5. What do you feel you have learned from the workshops?

6. Describe the characteristics of your teaching that make you a successful mathematics teacher.

7. Describe the changes you think you will make in your teaching as you implement the new programs.

8. What is your belief about how students best learn mathematics? Does it differ for different students and if so how?

9. What are your biggest areas of concern and how can these be best addressed?

Other comments:

Appendix D: Pre-Workshop Survey Results

Number of respondents giving each response is indicated to the left or beneath each response.

Survey

Thank you for taking the time to complete this survey. Your input is valuable and appreciated.
The survey is designed to collect data on teacher beliefs on the nature of mathematics teaching and mathematical learning and how that is reflected in or enhanced by classroom practice.
It is consists of 4 sections each containing 5 questions.

SECTION 1 Background Information

1. Indicate the Mathematics courses you are teaching this school year.

15	Math 30	10	Math 33	1	Math 24	6	Math 31	Math 33—prep
13	Math 20	5	Math 23	2	Math 14	5	M AP	Math 23—prep
9	Math 10	8	Math 13			4	Math 9	

2. Indicate the level of familiarity you have with the underlying philosophy of the Western Canadian Protocol for Mathematics.
   - 3 I am well aware of the philosophy.
   - 13 I am aware of the philosophy but am uncertain of details.
   - 4 I am not familiar with the philosophy.

3. Indicate the level of familiarity you have with the details of the general and specific learning outcomes of the Western Canadian Protocol for the grades you teach.
   - 8 I am aware of the general and specific learning outcomes for the grades I teach.
   - 0 I would feel comfortable explaining the general and specific learning outcomes to a colleague.
   - 12 I am not aware of the general and specific learning outcomes for the grades I teach.

4. Indicate the level of familiarity you have with the NCTM standards on Curriculum and Evaluation.
   - 2 I am well aware of the standards.
   - 13 I am aware of the standards but am uncertain of details.
   - 5 I am not familiar with the standards.

5. List the professional development activities that you have been involved in the last two years that have been useful to you and indicate what it was about the nature of these activities that made them useful.

10	MCATA Math Conferences	3	In school in-services	2	IB or AP Conference
1	Teaching Approaches	1	U of A Protocol Course		
1	Hands on Math-		Gr 9	3	Involvement with
1	Math 31 Workshop	3	Math Leader Symposium		publishers preparing
3	Calculator Workshop	3	Marking/Item Writing		materials
			Diploma Exams	4	No inservice

SECTION 2 Nature of Mathematical Learning

Circle the most appropriate choice.

1. Students learn best when they study mathematics that they can relate to a personal or social application.

Strongly Disagree		Neither Agree nor Disagree		Strongly Agree
1 ①	1 ②	0 ③	10④	9 ⑤

2. Almost all students can learn to think mathematically.

Strongly Disagree		Neither Agree nor Disagree		Strongly Agree
2 ①	3 ②	3 ③	10④	6 ⑤

3. Problem solving should be the main emphasis of mathematics instruction.

Strongly Disagree		Neither Agree nor Disagree		Strongly Agree
0 ①	2 ②	4 ③	10④	3 ⑤

4. It is important to master the basics before learning to solve problems.

Strongly Disagree		Neither Agree nor Disagree		Strongly Agree
1 ①	2 ②	2 ③	2 ④	7 ⑤

5. Indicate which of the following statements best describes the nature of mathematics from your perspective.

   15 Mathematics is a process of inquiry where patterns are generated and then added to the overall knowledge base. Mathematics is not a finished product

   4 Mathematics is a static, unified body of truths bound together by logic. Mathematics is therefore discovered not created.

   0 Mathematics is a defined set of rules and facts to be used by skilled craftsmen to accomplish a task. Mathematics is absolute and certain.

   Comments

   - Math is all of the above with some editing.
   - I think the patterns are there waiting to be discovered. If a theorem or proof changes it is because it was discovered wrong in the first place.
   - I feel if students can see the sequences and patterns it allows them to take ownership of their learning.
   - Problem solving needs to be very open to exploration and development, not just more practice.

SECTION 3 Teaching Strategies

1. Graphing calculators and computers are important tools in teaching problem solving.

Strongly Disagree		Neither Agree nor Disagree		Strongly Agree
1 ①	0 ②	4 ③	8 ④	6 ⑤

2. Children can develop problem solving skill by working together in small groups.

Strongly Disagree		Neither Agree nor Disagree		Strongly Agree
0 ①	1 ②	2 ③	11 ④	6 ⑤

3. Activity based experiences aren't worth the time and expense for what students learn from them.

Strongly Disagree		Neither Agree nor Disagree		Strongly Agree
5 ①	9 ②	4 ③	3 ④	0 ⑤

4. Students should have the opportunity to write about mathematical topics and their mathematical experiences.

Strongly Disagree		Neither Agree nor Disagree		Strongly Agree
0 ①	1 ②	5 ③	10 ④	4 ⑤

5. Based on your plans for the current semester estimate the amount of emphasis you place on each of the following activities. You may choose to do this for an individual class or all classes as a collective. If you choose a particular class indicate the kind of class. (i.e. Math 13, Math 9) and if you choose collective indicate the grade levels.

CLASS _____ COLLECTIVE (grade levels) _____

	③ No Emphasis	② Minimal Emphasis	① Significant Emphasis
Lecture	0	4	17
Discussion	0	7	13
Student reports/projects	5	10	6
Research (library or interment)	12	5	3
Students using graphing tools	1	4	14
Students using computers	7	8	5
Guest speakers	14	4	0

	③ No Emphasis	② Minimal Emphasis	① Significant Emphasis
Field trips	15	1	0
Small group work	0	9	8
Students using manipulatives	8	8	2
Discussion of homework	0	2	17
Seatwork	1	5	14
Tests/quizzes	0	0	16
Performance Assessments	1	8	8
Students at the board	5	7	4

Other Activities (specify)

- Project work applying work covered; working as partners on a daily basis; error analysis

SECTION 4 Professional Growth

1. I feel supported by colleagues to try out new ideas in teaching mathematics.

Strongly Disagree		Neither Agree nor Disagree		Strongly Agree
1 ①	0 ②	4 ③	3 ④	11 ⑤

2. Mathematics teachers in my school meet regularly (formally or informally) to share ideas and materials.

Strongly Disagree		Neither Agree nor Disagree		Strongly Agree
0 ①	1 ②	2 ③	5 ④	10 ⑤

3. I have time during the school week to work with my peers on mathematics curriculum and instruction.

Strongly Disagree		Neither Agree nor Disagree		Strongly Agree
6 ①	5 ②	3 ③	4 ④	1 ⑤

4. It is more important to receive continued training in mathematics than it is to receive training in effective ways to teach and manage mathematics classrooms.

Strongly Disagree		Neither Agree nor Disagree		Strongly Agree
4 ①	7 ②	4 ③	4 ④	1 ⑤

5. I regularly read journals and publications about Mathematics and Mathematics teaching.

Strongly Disagree		Neither Agree nor Disagree		Strongly Agree
1 ①	3 ②	6 ③	4 ④	5 ⑤

Thank you for you participation. Please feel free to provide any comments.

Appendix E: Post-Workshop Survey Summary

1. Describe the philosophy and objectives of the Western Canadian Protocol as you understand it.

Response 1
- to present mathematics in a meaningful manner.
- to help students to make better connections in their mathematical learning
- to develop concepts from real world applications and then to use -the concept in other real world applications
- to have students more active in their learning - doing experiments, labs, etc.
- to use technology as a tool in solving problems
- pure - represents the theory of math
- applied - more tactile, for the application of math concept
- Wow! - in 25 words or less?? The philosophy of the WCP stresses things like cooperative learning, group decision making and perhaps most importantly, mathematical communication skills
- try to get all of Western Canada on same Math program to enable students to transfer between province
- bring in more technology to support businesses
- to learn mathematics by putting it into a practical context
- to have a common Math framework across Western Canada, including common textual material. This enables mobility of families with constant Math curriculum for students
- use of technology and more hands on
- using more manipulatives, hands on, and use more technology

2. Describe the characteristics of a mathematics classroom you envision based on your understanding of the new APPLIED curriculum.

Response 2
- to have students working in pairs(groups) on an experiment or laboratory assignment - teacher as a facilitator
- to have access to supplies - class sets of compasses, protractors, rulers, scissors, meter sticks, yard sticks, etc.
- students actively involved in learning
- discovery and lab approach used rather than lectures (computers, etc.)
- probably hand-on activities, less algebra
- definitely more technology and CTS modules
- more hand on material - should be easier at the grade 10, 11 level for the mathematically theory-challenged students
- good work
- student centered

- students communicating with one another about problems in math
- more hand on activities
- more "guess and check"
- more technology ie. spreadsheets, graphing, calculators
- different approach
- more manipulatives, computer, calculators, noisier, more student directed, control a question
- more participation from the student, do in class project with manipulatives, use of technology (computers, spreadsheet, graphing, calculators). More communication between the student and more noise

3. Describe the characteristics of a mathematics classroom you envision based on your understanding of the new PURE curriculum.

Response 3
- similar to #2 above but not to the same degree-the course is still fairly algebraic/theoretical based, therefore the time to do as many experiments/laboratories is limited
- more theoretical - traditional approach
- calculators and computers used
- level of math higher than previous
- I don't think there will be as great a change at this level as there will be at the APPLIED level
- more implementation of technology - especially computer labs for the use of spreadsheets - rest of curriculum pretty much the same as was
- investigating math topics using graphing calculators
- learning through investigations
- more technology-spreadsheets, graphing, calculators
- more hands on
- somewhat the same as today, with a lot more emphasis on technology
- use more of the technology but not much change from what is now

4. Which of the activities that we did in this series of workshops were most useful to you? Why?

Response 4
- the workshops in themselves were good as it forced me to look at the new curriculum in some detail
- the discussions with the other members of the class were invaluable
- the three activities-toothpicks, balls, solar system-started to give us some ideas of the "things" that can be included
- looking at some classroom activities that we may use
- practical

- the activity in which we used the sticks to create patterns was interesting because it was one of the few that I had not done before. I enjoyed the map activity

Response 4 continued

- the sharing of ideas from all the other math teachers from different schools
- the first activity in which we were to look through the curriculum and figure out what was to be taught in each course. It has help me understand what will be in each course and how to approach it
- analyzing the curriculum
- networking with other teachers
- the project - an in depth study - was very useful
- the activities we tried that we could use in a classroom - very practical - something useful
- I missed the last 2 and so it is hard. The first 2 were excellent
- everyone of them because it give me great ideas and a very big boost of energy to try something new

5. What do you feel you have learned from the workshop?

Response 5

- that this curriculum is different from what we presently have
- that my role as a teacher will need to shift more from presenter to facilitator
- that I will need to spend more hours in trying to prepare to teach these courses than I had to spend in the last change
- that I need to learn how to do spreadsheets
- that I need to make more budget demands for mathematics at my school
- more about the WCP in general
- Some of the activities were interesting, but it would be entirely possible to lose the objective in the activity. Activities have to be carefully planned
- get organized now! Share more with others (as we do now) Greater Edmonton Math Teachers
- understand what will be in each course and how to approach it
- many things
- some class projects, overview of the curriculum
- the content of the curriculum and activities
- I feel I have learned to not be afraid of this change in curriculum. I have some knowledge of the philosophy (where they ant us to go with it) and also what the content of the curriculum is to be

6. Describe the characteristics of your teaching that make you a successful mathematics teacher.

Response 6

- I find this one hard to answer as I know that I am not successful with some of my students. I will answer this on the premise that I am successful with some of my students
- for every class, I have a "plan" but as the class unfolds, I have the knowledge and experience to "think on my feet" in order to make the class as productive as I can but keeping most of the students on task
- I try to make the lesson part as short as I can (but it needs to cover all the basics plus some higher - thinking aspects) so that students have a maximum time to work on their assignments
- knowledge of math
- good relationship with students
- flexible
- I like working with students in mathematics, and I believe students like work with me. My powers of explanation are very good. I am able to "think through" situations with students. I am able to approach mathematics using different strategies and I encourage students to do the same. I use technology a great deal, and can support the mathematics using technology
- organization
- having a teacher - network to bounce ideas off
- organized
- math background
- communication skills
- I try to make math easier for the students, to show them where they go wrong so they know what to look out for
- I try to relate math to their everyday life or future career
- I am always there for help. Students feel comfortable coming to me for Math help
- I have a sense of humor
- my ability to communicate and create a class to become a team. My role as coach, their role as players, success being a team goal
- my passion of math is transmitted to my students. I make them feel as if we were a team and we have a mission, the only way to arrive at term is communication, respect, creating, taking advantage of every opportunity. If we stand together we stand a chance of succeeding!

7. Describe the changes that you think you will make in your teaching as you implement the new program.

Response 7.
- to be more of a facilitator than a presenter
- to use technology especially spreadsheets
- to use activities, experiments, laboratories, more
- in preparing lessons, use more resources than just the textbook, unless the textbook has everything that I need
- prepared to try more "open" classroom - lab approach for applied
- not withstanding the recycling on content - the return of vectors, matrixes - a major change, especially at the applied level, will be that more materials are required. Computers (for spreadsheets) could be a problem
- get more ideas from the CTS teachers re: spreadsheets, designs, layouts for Applied
- use more computers/calculators
- more student directed activities
- lab like situations
- less lecture
- I will make it more lab (science) oriented
- I will try to bring more of the community/real life into the Math classroom
- I will try to start each lesson off with something this will catch the interest of the student. That the approach is different
- technology, less teacher, more student directed
- manipulatives
- already started the change. I started to use more technology, like Power Point, and more manipulatives hand on project

8. What is your belief about how students best learn mathematics? Does it differ for different students and is so how?

Response 8
- students learn mathematics best when they like mathematics which is based on their previous encounters with mathematics
- they know where they are headed after high school
- they pay attention to lessons taught, do their homework and have a good feeling about their success
- they have the opportunity to be active in their learning
- they have some idea where the concept can be used in real life
- students learn mathematics in different ways depending if they are more science/math oriented or social/english oriented - left brain/right brain dominant
- their previous experiences, maturity, future goals
- students learn in many different ways. Some like lecture style - some need to be doing. Probably many need some of both. I hope that the WCP will address both needs
- there are many different learning modes. It seems to me that the best students approach math as a foreign language - they simply speak it, and understand it. They are able to think on mathematical terms. Numbers and symbols are a way of life!!
- they have to be interested in it first. A lot of students learn math by understanding how things work, some seem to only learn by rote
- students learn by doing, but they need guidance and direction as well
- some are visual learners, auditory learners, hands on learners. Some learn best on their own, others in group situations. They all learn differently. I think they learn Math best with many different approaches. That is why Math teachers need to be more open minded and willing to try new ideas
- I believe students must have the basics, must be able to work without a calculator, and then can we move. Students then can move on to deeper concepts and HMA concepts where manipulatives and technology will enhance the program
- visual, hands on and most important of all build their confidences by the basics. Every students learns by visual and hands on but some of them needs more then that, and that's where technology may bring them to the next level

9. What are your biggest areas of concern and how can these be addressed?

Response 9

- how to use a spreadsheet for the concepts in the curriculum
- need in servicing - take a course, have some show me
- teachers willingness and knowledge about how to teach Applied
- my biggest area of concern is this - how well will there "new" learning practices equip students for university? Will students be any better prepared for the "real world" then they now are?
- making sure that the resources we pick are conducive to learning - ex. M matter series was very poor so don't want to make that mistake again.
- would probably purchase a sample of each resource and use both as a guideline until the teachers decide which is the better resource
- lack of ready resources
- money for computers/calculators
- the technology issue - every student in applied will require a graphing calculator. In applied - the "guess and test approach" with technologies (spreadsheets). I don't think the students will gain very much Math knowledge with this approach. I think they will become frustrated
- too much technology, student not having a solid foundation from which to work from
- technology to soon! Build the confidence with manipulatives and the old fashion way, pencil, eraser and paper. We must build a foundation to jump from
- time in order to make a textbook selection
- to prepare a course outline when we have received no recommended percentages for each of the units
- to prepare lesson plans in which you need materials to do an experiment, laboratory, etc. including the booking of the computer lab for the day you need it - be well enough prepared that in case the computer is not available or not working that you carry on with meaningful work
- release time given by the school or the district
- the timing that we have been given for implementation - if all of the major decisions of implementation, textbook selection, etc would have been ready a year ago from when they have or will be decided, then this set of workshops plus other inservices would have much more production as we have spent more hours trying to find out what is happening, when it is happening, etc - most teachers are going to try to "implement" this curriculum with lots of "baggage" that they have had no control over

- Alberta Education has to have a serious look at how they want curriculum to be implemented especially when there are major changes from what currently has been happening

Other Comments

Responses

- this series of workshops has certainly been helpful especially since you had a connection with Alberta Education and could keep us updated and/or taking our concerns back to Alberta Education. It could have been more "productive" if we didn't have to spend so much time with dealing with the frustrations that it has caused for us - for example, if we would have had the approved resources, we could have each prepared some lesson plans, went to the computer laboratory to use a spreadsheet for a particular unit, etc, - we could have taught each other and been exposed to other ways of presenting material that we may not have thought of or do not have the background and or confidence to try an innovative idea
- I admire and complement you on organizing and presenting these workshops as I would say this was not the ideal situation - in fact I hope that the way this curriculum has been implemented never happens to anyone at any grade level and/or subject area and particularly, at the senior high mathematics level. All the best in trying to use this in your thesis.
- it is always difficult to attend workshops after a day at work or end of the week. My creativity factor is not nearly as good. From my perspective, we could have saved time by doing less group work though this would have meant you doing more demonstration
- my best way of learning is I'm totally a visual learning
- will university ultimately adapt to "new" methods of instruction? If they do, I believe it will take many years for the changes to work through the system
- letting parents, counselors, community know about the changes and how can we sell the Applied program. This will come as the resources are intact and as the post secondary institutions come on board esp. U of A.
- the workshops were excellent! Thanks for letting me in . P.S. I'd like to read thesis when you're done.
- I thank you for your time, and apologize for missing the last 2 classes. Just not enough time in my week any more.
- I thank you for this opportunity and your time, it was very informative for me and energy booster. Hopefully will have more of these. Thank you and good luck with your thesis

Appendix F. Volume Activity

-A Collection of Connections, Sigurdson, S. & Kieren, T., CMASTE, 1997.
(Used with permission)

The Volume of a Sphere

General Question

The textbook tells us that the formula for the volume of a sphere is $\frac{4}{3}\pi r^3$. Why is this a reasonable formula? What meaning does this formula have for us? We know what the formula for the volume of a sphere is but do we understand it? Today we will take a procedure from science and use it to help us understand some mathematics. Usually we use mathematics to help understand science but today we have the reverse. Why is the formula for the volume of a sphere $\frac{4}{3}\pi r^3$?

Discussion questions

- What is the formula for the volume of a cube of side s?
- What is the formula for the volume of a cylinder of radius, r, and height, h?
- Describe what r^3 means physically.
- What is the area of a circle?
- Do you have a way of remembering the area of a circle?

Preliminary Activities

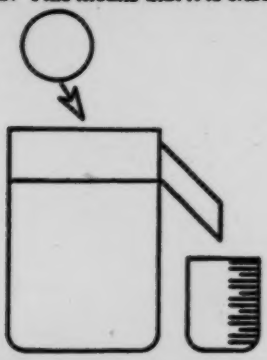
Surface area of a sphere

Earlier in this Collection we found out how many stars we could see. We used the formula for the surface area of a dome (half of a sphere). How do you think that was calculated? The formula for the surface area of a half sphere is $2\pi r^2$. Let's compare it to something you already know. Imagine you had an elastic circular disk with radius r and it could be changed into half of a sphere by blowing air on it. The surface area of that disk when it is flat is πr^2. The surface of the curved dome should be about double the area of the flat disk upon which it rests. Although this is not a proof of the surface area of a hemisphere formula, it makes the formula reasonable. Using the same thinking, the surface area of a sphere is 4 times the area of the circle through its centre, $4\pi r^2$.

The area of a parallelogram can be compared to a rectangle on the same base and the same height. In geometry, it is often useful to compare one figure to another. In this lesson we will compare the volume of a sphere to the volume of the cube and the cylinder into which the sphere could be placed exactly.

A lesson from science

We can determine the volume of the sphere by water displacement. The science laboratory may provide us with a displacement jar. We fill the jar so that water flows out of the jar. This means that it is exactly full (as in the picture).



When the sphere is lowered into the water some water will spill out of the spout. The amount that spills out the spout is exactly equal to the volume of the sphere. Measuring the volume of water that collects in the catch beaker is a way of determining the volume of the sphere. If 20 mL (millilitres) of water is collected the volume of the sphere is 20 cc (cubic centimetres).

Discussion questions

- What shape does the object have to be for this to work? (Shape is irrelevant. The method is most often used for irregularly shaped objects.)
- What if the object floats?
- What if the object sinks to the bottom?
- If our first object is in the displacement jar can we put another object in?
- What condition must be met for the displacement test to work?

In order to understand our lesson students must be convinced that no matter how far down the sphere is in the displacement jar, the volume of water displaced will be equal to the volume of the sphere.

Verifying volume by water displacement

To help students see that the displacement method of finding volume actually works, the teacher can demonstrate the method using an object of known volume. Connecting, for example, eight, one cubic centimetre blocks together would create an object of known volume and could be used for the demonstration. To show that the shape of the object is irrelevant, connect the blocks together to form different shapes and repeat the demonstration.

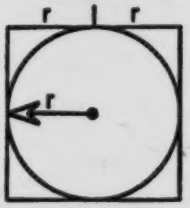
Answering the General Question

If we simply wanted to find the volume of the sphere we could displace water and have our answer. However, we want to find the formula for volume. We do this by

comparing this volume to the volume formula for something we already know. We have two such objects, a cube and a cylinder.

A cube

Imagine a cube just large enough so that our sphere fits into it exactly. The dimensions of the cube will be 2r x 2r x 2r. The diagram of a circle in a square illustrates the point.



The diameter of the circle is 2r. Therefore, each side of the square is 2r. The volume of our cube is $8r^3$ and we can easily see that the volume of our sphere is less than this. A reasonable guess is that it is about half this amount, or $4r^3$. That is, the volume of the cube outside the sphere is about equal to the volume that is inside the sphere.

A cylinder

Now let us imagine our sphere fitting exactly into a cylinder which is the same height as the sphere. When we look at this from the top of the cylinder the sphere touches at all points of the cylinder. It is a perfect fit. However, when we look at it from the side, if we could see through the cylinder, the picture would look like this.

The sphere would not be touching the cylinder in any of its corners. We need to recall that the formula for the volume of a cylinder is $V = \pi r^2 h$. In this case $h = 2r$ (the diameter of the sphere) so the formula becomes $2\pi r^3$. So now we can see that the formula for the volume of a sphere is less than $2\pi r^3$ because the sphere does not entirely fill the cylinder. A good guess might be one-half of $2\pi r^3$ or πr^3.

Looking back to our earlier guess of $4r^3$, based on the volume of a cube, we should notice that the estimate of πr^3 is about the same since this is about $3r^3$ (because $\pi = 3.1$). We might also reasonably guess that πr^3 is in the formula. The constant π does not appear in the cube formula since the cube is not in any way in the shape of a circle while the cylinder is.

These comparisons have yielded two insights:

i) The formula for the volume of a sphere is close to $4r^3$ and πr^3.

ii) The formula probably contains πr^3.

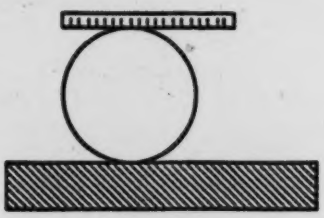
A measurement procedure

We now know that the formula for the volume of the sphere is some number "K" times r^3. We are also reasonably sure that π is part of the number K. We are going to use

our science measurement to make a reasonable guess at that number. We know that it must be fairly close to 3 or 4.

Each group is given a different ball. All, however, need to fit in the displacement jar. If they sink on their own accord so much the better. Otherwise we will have to find ways to submerge them. However, before students find the displacement they should figure out the volume of the cube and cylinder that is related to their ball.

Exact measurements are a must, certainly to the nearest millimetre. One way of measuring the diameter of the sphere is to wrap adding machine tape around it and simply measure across the edges of the tape. Placing a ruler on top of the ball parallel to the table top is another way.



The students have four tasks:

1. Find the radius of the sphere.
2. Find the volume of the related cube.
3. Find the volume of the related cylinder.
4. Take the actual sphere and see how many cubic centimetres it displaces.

The displacement activity

As each group submerges its sphere in the displacement jar, the teacher will need to supervise the activity. The displacement jar must be full and the catch beaker must be empty. The tricky part is to fully submerge the sphere. If the sphere is heavier than water this is easy. If it is lighter and floats, a device will need to be used to push the sphere down so that it just submerges. If the pushing device goes into the water, it will displace water giving inaccurate measurements.

The larger the ball is the less percent error there will be. Small balls give large errors. The line of best fit method (given below) will make it clear when one of the measurements is erroneous This measurement should be excluded.

Using a table

In the chart we will include the radius, the volume of a cube, the volume of a cylinder, and displacement. The latter three numbers will give us a basis for comparison. Since the volume of the sphere is probably going to be some multiple of r^3, we will also put that number in a sixth column. This is a calculation based on the radius.

1	2	3	4	5	6	7 optional column
Name of group	radius (least to greatest)	volume of cube	volume of cylinder	displacement	value of r^3	displacement divided by r^3

Once all the groups have contributed their numbers to this chart, the class can proceed to make a graph of r^3 on the x-axis and the displacement on the y-axis.

Teaching suggestion

The teacher should supervise the filling in of the chart. Once this is done, students can be assigned the task of drawing the graph of the displacement against r^3. Using the line of best fit method, they will try to determine the line on the graph. Those points that fall far from the line may be due to error and may be eliminated.

A discussion of error is a chance to reflect on the displacement activity and to focus on the physical nature of the measurements. This, of course, contrasts to the error free use of mathematics that occurs in typical mathematics classes. The discussion of error brings out the real world aspects of this activity.

Determining the equation

Points on the line can be used to determine the equation for the relation:

$$D(\text{isplacement}) = K \times r^3$$

The value of K should be close to 4. The actual value is $\frac{4}{3}\pi$ which is about 4.2. The discussion of the constant K should centre on whether $4r^3$ or $\frac{4}{3}\pi r^3$ is closer.

The formula of the volume of a sphere is $\frac{4}{3}\pi r^3$. Students can then be asked to find out what fraction the sphere is of the cube and what fractions it is of the cylinder. It is $\frac{1}{2}$ of the volume of a cube and $\frac{2}{3}$ the volume of the cylinder. A discussion of these results should help to confirm that the formula for the volume of a sphere is reasonable.

An alternative method of data analysis

A simpler, but less accurate way, of finding K is to create another column for "the displacement divided by r^3". The numbers in this column should all be close to 4.2, that is $\frac{4}{3}\pi$.

Discussion questions

This activity is not a proof of the formula but the activity shows how the volume of a sphere is related to the volume of a cube, the volume of a cylinder and πr^3.

Materials

Displacement jar, water, spheres of various sizes and grid paper.

Modifications

If a displacement jar is not available, a beaker or a large graduated cylinder may be used. In this case, the volume of the sphere is the difference between the volume reading while the sphere is submerged and the volume reading before the sphere is submerged in the water.

The problem in this activity is accuracy. However, the main idea is to have students thinking of the cube, cylinder, and the sphere in comparison terms and also identifying r^3 as the key variable in the change of volume with π being a unique and important constant.

The whole activity might be done as a teacher demonstration. In this case the students miss the multiple calculations and the discussion of error and how it arises. The teacher may point out that error in using a formula arises when the measurements for a variable used in the formula contain error.